

$U_A(1)$ anomaly and η' mass from an infrared singular quark-gluon vertex*

Christian S. Fischer^{1,2}, Reinhard Alkofer³, and Richard Williams²

¹GSI, Darmstadt, Germany; ²TU Darmstadt, Germany; ³Graz University, Graz, Austria

The $U_A(1)$ problem of QCD is inevitably tied to the infrared behaviour of quarks and gluons with its most visible effect being the η' mass. A dimensional argument of Kogut and Susskind showed [1] long ago that the mixing of the pseudoscalar flavour-singlet mesons with gluons can provide a screening of the Goldstone pole in this channel if the full quark-quark interaction is strongly infrared singular as $\sim 1/k^4$. A few years later Witten and Veneziano [2, 3] proposed their solution of the problem by considering an expansion of QCD in N_f/N_c , where N_f and N_c are the number of flavours and colours respectively. They showed that the correct pattern of the $U_A(1)$ -symmetry breaking can be obtained if the anomalous mass of the η' , m_A^2 , is related to the topological susceptibility of the theory through the Witten-Veneziano formula

$$m_A^2 = 2 \frac{N_f}{f_0^2} \chi^2, \quad (1)$$

which includes the pion decay constant $f_0 \simeq 93\text{MeV}$. These two ideas, the Kogut-Susskind szenario of an η' driven by confinement and the Witten-Veneziano formalism are certainly not mutually exclusive. On the contrary, the same topologically non-trivial gauge field configurations could be responsible for both confinement and the resolution of the $U_A(1)$ -problem.

In our work [4] we investigated the Kogut-Susskind idea taking into account nonperturbative information on the infrared behaviour of the gluon propagator and the quark-gluon interaction. We employ solutions to the Dyson-Schwinger equations for the propagators of QCD and the quark-gluon vertex to determine the anomalous mass of the η' in the chiral limit. To this end we evaluated the so called diamond diagram, given in Fig. 1. It provides for non-

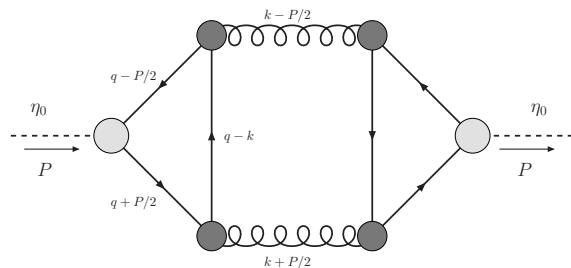


Figure 1: The diamond diagram $\Pi(P^2)$.

perturbative contributions to the meson masses which are only present in the flavour-singlet channel. The diagram is

m_ρ MeV	m_A^2 GeV ²	χ^2 (MeV) ⁴	θ	m_η MeV	$m_{\eta'}$ MeV
747	0.56	169	-23.2	479	906

Figure 2: Numerical results.

non-vanishing in the chiral limit if and only if it carries the $1/p^4$ -singularity in both momenta of the gluons. These singularities, however, are not generated by the gluons alone as assumed in the Kogut-Susskind model. Instead by now we know that the momentum dependence of the gluon propagator function is given by $\frac{Z(p^2)}{p^2} \sim (p^2)^{-1+2\kappa}$ with a positive anomalous dimension κ . However, from the analysis of [5] we know that the quark-gluon vertex, $\Gamma_\mu^\alpha = \frac{\lambda^a}{2} \Gamma_{\mu,\alpha}$, should have the behaviour: $\Gamma \sim (p^2)^{-\kappa-1/2}$. Putting dressed propagators and vertices together we arrive at the necessary $\Gamma^2 Z \sim 1/p^4$ singularity as necessary for the Kogut-Susskind mechanism.

Our numerical calculation of the diagram Fig. 1 and subsequent $\eta - \eta'$ -mixing have been determined with input for the gluon propagator and the quark-gluon vertex as described in detail in [4]. The parameters are chosen such that the mass of the pion $m_\pi = 135\text{ MeV}$ has been reproduced. Our results are shown in Fig. 2. They confirm the analytic analysis: we find a topological mass of the η' and a topological susceptibility χ^2 in agreement with corresponding results from lattice gauge theory [6]. Taking into account $\eta - \eta'$ -mixing we find values for the mixing angle θ and the masses of η - and η' -mesons that are of the same order as the experimental values. This gives us confidence that the mechanism as such is correct and at work.

References

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