

# Finite Volume effects from Renormalization Group flows

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In recent years, the accuracy of lattice QCD calculations has increased considerably, and simulations with dynamical fermions have reached rather low pion masses. In agreement with observations, a famous result by Lüscher [1] predicts that finite volume effects are strongly suppressed for large pion masses, but should become much more important for small, realistic values. For this reason, there is an interest in the calculation of such effects by other means, in addition to the direct investigation on the lattice. The principal method used for this purpose is chiral perturbation theory (chPT), the low-energy effective field theory of QCD.

The application of chPT to finite-volume QCD is almost as old as the theory itself [2]. In an effective field theory, unknown higher-momentum effects are encoded in the values of the low-energy couplings (LECs). These values in the low-energy effective Lagrangian remain the same when introducing a finite temperature, but not necessarily when the underlying QCD is put into a finite Euclidean volume. While the boundary conditions for quark fields must be anti-periodic in the time direction, they can be chosen freely in the space directions. In [2], it is argued that due to the symmetry between spatial and temporal directions, the LEC remain unchanged for a choice of anti-periodic quark boundary conditions in the high-energy theory. However, they might change for periodic quark boundary conditions.

In order to investigate the effects of quark boundary conditions, we turn to the quark-meson model, in which constituent quarks are coupled to a linear  $\sigma$  model [3]. In this model system, chiral symmetry is broken dynamically through the formation of a quark condensate. We use non-perturbative Renormalization Group (RG) flow equations to systematically integrate out quantum fluctuations and obtain the effective action. Since the bare couplings of the model are fixed at some UV scale in order to reproduce observables for infinite volume, only the behavior in finite volume is predicted [3, 4].

In accordance with the predictions of [2], we find agreement between chPT and our results for anti-periodic boundary conditions, but qualitatively different behavior for periodic boundary conditions (Fig. 1). In general, finite volume effects are much smaller for periodic boundary conditions, the usual choice in lattice calculations.

The chiral phase transition temperature in finite volume is affected by the boundary conditions as well (Fig. 2), even for volumes  $L \simeq 2 - 2.5$  fm [5]. While the absolute value of the transition temperature in the chiral limit is too low compared to lattice or QCD RG investigations [6], we expect qualitatively similar behavior on the lattice.

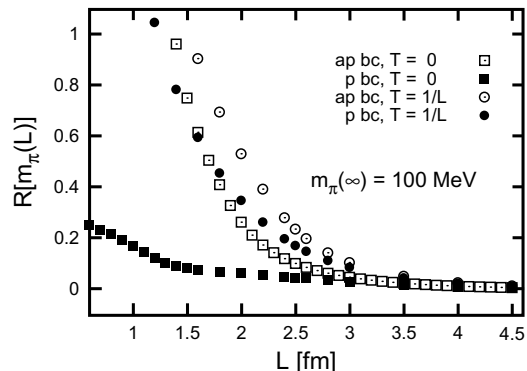


Figure 1: The pion mass shift  $R[m_\pi(L)] = (m_\pi(L) - m_\pi(\infty))/m_\pi(\infty)$  in a finite volume  $V = L^3 \frac{1}{T}$ , as a function of  $L$ . Results are for anti-periodic (solid) and periodic (open symbols) quark boundary conditions, for fixed  $T = 0$  (boxes), and for  $T = \frac{1}{L}$  varying with the volume size (circles), for a pion mass  $m_\pi(\infty) = 100$  MeV.

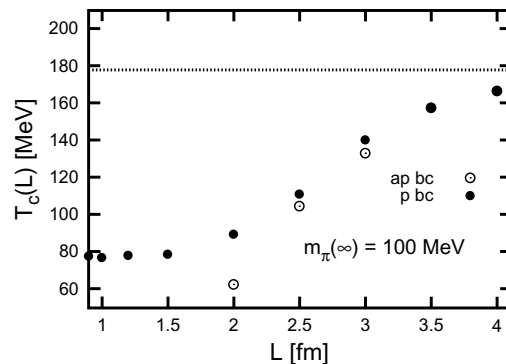


Figure 2: Chiral phase transition temperature in a finite volume  $V = L^3 \frac{1}{T}$  as function of  $L$ , for periodic and anti-periodic quark boundary conditions, for a pion mass  $m_\pi(\infty) = 100$  MeV. The crossover temperature is defined via the minimum of the  $\sigma$  mass.

## References

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\* supported by GSI