Scattering of residual gas atoms by fast HCI beams

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The scattering of fast ion beams, circulating e.g. in a synchrotron, with the residual gas can lead to desorption when the scattered ions hit the aperture limiting devices of the structure [1]. Here, the impacts of recoiled residual gas particles may be an additional source for desorption. We are thus interested in the scattering of residual gas atoms or molecules by passing fast highly charged ions, focusing on the example of 1 GeV/u U^{92+}.

To make an estimate for the related cross section and the acquired recoil energies we treat the problem as elastic scattering between the nucleus (with charge $Z_R e$) of a residual gas atom and the projectile nucleus ($Z_P e$), neglecting the energy and momentum transfer to the bound electrons of the residual gas atoms or molecules. A classical description with classical trajectories can be employed because the Coulomb parameter $\eta = Z_R Z_P e^2/(4\pi\epsilon_0\hbar c\beta) = Z_R Z_P \alpha/\beta$ is for the envisaged energies and projectile charges ($\beta = v/c = 0.87$, $\gamma = 2$, $Z_P = 92$) about unity, or even larger for residual gas atoms heavier than hydrogen [2, 3]. As shown in [4], magnetic field effects and retardation corrections are rather small for relativistic (heavy) ion collisions at such energies ($\gamma = 2$) and will be neglected.

We then end up with classical scattering with an effective interaction potential $V(r)$ which accounts for the screening of the nuclei by the bound electrons. This can be treated by relativistic classical mechanics. Here closed expressions can be obtained for arbitrary spherically symmetric interactions $V(r)$ [5], if one collision partner remains at rest. As a next step of approximation we thus consider the scattering event in the frame where the heavy projectile (^{238}U) is (initially) at rest and treat it as infinitely heavy ($m_P \to \infty$). Replacing the effective $V(r)$ by the Coulomb potential, i.e. $V = \kappa/r$ ($\kappa = Z_R Z_P e^2/(4\pi\epsilon_0)$) and taking the screening into account only by restricting the allowed impact parameters b to values below a typical screening length $\lambda \approx 10^{-10}$m, the scattering angle $\theta(b)$ can be given explicitly [5] by

$$\theta(b) \;=\; \pi - \frac{2}{\zeta(b)}\mathrm{arccot}\left[\frac{b_0}{b\,\zeta(b)}\right],$$

where $b_0 = \kappa/(\gamma m_R c^2\beta^2) = Z_R Z_P \alpha\hbar/(\gamma m_R c\beta^2)$ and $\zeta^2 = 1 - \beta^2 b_0^2/b^2$. In the present case $\gamma = 2, \beta^2 = 3/4, Z_P = 92$ we get (with $m_R = A_R m_{\mathrm{u}}$) $b_0 = (Z_R/A_R)\;0.953 \times 10^{-16}\mathrm{m} \approx 10^{-16}\mathrm{m}$. For impact parameters b between $\approx 10^{-14}$m (sum of nuclear radii) and $\lambda \approx 10^{-10}$m (screening) we have $b_0/b \ll 1$, hence $\zeta \approx 1$. For the angle $\theta(b)$ and the cross section $d\sigma/d\Omega = [b(\theta)/\sin\theta]\,|db/d\theta|$ we thus arrive at

$$\theta(b) \;\approx\; 2\frac{b_0}{b} \;\ll 1, \qquad \frac{d\sigma}{d\Omega} \;\approx\; \frac{4b_0^2}{\theta^4}\,.$$

A Lorentz back-transformation (with $-\beta, \gamma$) to the lab-system, where the residual gas atoms are initially at rest, yields the scattering angle θ' and cross section

$$\tan\theta' = \frac{\sin\theta}{\gamma(1-\cos\theta)} \approx \frac{2}{\gamma\theta}\,, \quad \frac{d\sigma'}{d\Omega'}(\theta') \;\approx\; \frac{\gamma^2 b_0^2}{\left(\frac{\pi}{2}-\theta'\right)^3},$$

that is, the residual gas atoms are mainly scattered with angles close to $\pi/2$, i.e. perpendicular to the projectile beam, with a momentum (in the lab-frame) of

$$p'^2 = p^2\sin^2\theta + \gamma^2 p^2(1-\cos\theta)^2 \approx p^2\theta^2 = \gamma^2 m_R^2 c^2\beta^2\theta^2$$

i.e. $p'/m_R c = \gamma\beta\theta \ll 1$. This results in the corresponding non-relativistic recoil energies E_{kin} of the scattered residual gas particles (per u)

$$\epsilon = \frac{E_{\mathrm{kin}}}{\mathrm{u}} \approx \frac{1}{2}m_u v'^2 \approx \frac{1}{2}m_u c^2\gamma^2\beta^2\theta^2 \approx 1\mathrm{GeV}\theta^2\,.$$

These energies are varying from sub-eV for collisions with b around the screening length λ ($\theta \approx 10^{-6}, d\sigma'/d\Omega' \approx 10^{-14}$ m^2/rad) up to values of a few 100 keV for collisions where the nuclei almost touch ($\theta \approx 10^{-2}, d\sigma'/d\Omega' \approx 10^{-26}$ m^2/rad). As an estimate for the expected number of residual gas particles dN scattered per unit time with a recoil energy within $[\epsilon, \epsilon + d\epsilon]$ we get (since $\epsilon \propto \theta^2$)

$$\begin{aligned} dN(\epsilon) \;&= dN(\theta(\epsilon)) = J\frac{d\sigma}{d\Omega}(\theta)2\pi\sin\theta d\theta \approx J8\pi b_0^2\frac{\theta d\theta}{\theta^4} \\ &\approx 2\pi b_0^2 m_u c^2\gamma^2\beta^2\,J\,\frac{d\epsilon}{\epsilon^2} \approx\; 10^{-25}\,\frac{J\,d\epsilon}{\epsilon^2}\;\mathrm{keV\,m^2} \end{aligned}$$

per one passing heavy ion (i.e. a 1 GeV/u U^{92+} ion). J is here the current density of the residual gas atoms in the frame where the heavy ion is (initially) at rest.

References

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