

Relativistic Fermi-liquid theory of dense matter

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Low temperature many-fermion systems can be described by Landau Fermi-liquid theory [1]. The parameters of the theory can be determined either empirically or by direct calculation. However, for gauge theories (QED/QCD) there are problems. We explore the Fermi-liquid properties of relativistic electron and quark systems at small temperatures and high densities. Polarization contributions to the quasiparticle effective interaction are computed in the random-phase approximation¹.

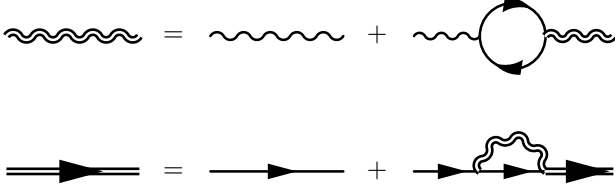


Figure 1: Effective interaction and quasiparticle propagator in random-phase-approximation

At low temperatures longitudinal gauge bosons acquire a mass in the static limit, the Thomas-Fermi mass M_{TF} . On the other hand, the transverse ones remain massless:

$$\Pi_L(k_0, \mathbf{k}) = -M_{TF}^2 - i \frac{\pi M_{TF}^2 k_0}{2v_f |\mathbf{k}|} \coth \frac{k_0}{2T} \quad (1)$$

$$\Pi_T(k_0, \mathbf{k}) = +i \frac{\pi v_f M_{TF}^2 k_0}{4 |\mathbf{k}|} \coth \frac{k_0}{2T}. \quad (2)$$

Here v_f denotes the Fermi-velocity of the particles.

Due to the static screening in the longitudinal part, the exchange of a longitudinal boson is infrared finite. Conversely, the transverse part diverges at zero temperature due to the lack of static screening. However, at finite temperatures the transverse channel is screened by the imaginary part in (2). This dynamical screening renders the transverse contribution finite. To leading order in T and $\omega = (p_0 - \mu)$ one finds for the fermion self-energy

$$\text{Im}\Sigma^+(\omega) = -\frac{e^2 v_f \omega}{24\pi} \quad (3)$$

The corresponding single-particle strength of the quasiparticle is given by:

$$Z^{-1} = 1 - \left. \frac{\partial \text{Re}\Sigma^+(\omega)}{\partial \omega} \right|_{\omega=0} = 1 - \frac{e^2 v_f}{12\pi^2} \ln cT, \quad (4)$$

where c is a constant. Thus, at temperature strictly zero, the Z -factor and the discontinuity at the Fermi surface vanish. Furthermore, the Landau parameters diverge. Consequently, such a system is not a normal Fermi liquid [2]. However, at finite temperatures the singularity is washed out and the system may be described as a Fermi liquid.

At non-zero temperature also the Landau parameters are finite. For the one-photon-exchange interaction of Fig. 2 one finds [3]:

¹We present only the QED results here. The QCD results are obtained by a straightforward generalization.

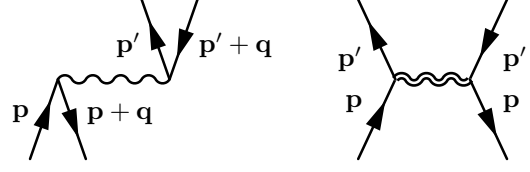


Figure 2: Interaction processes on the Fermi-surface. The interaction is particle-hole reducible in the u-channel (right) and remains irreducible in the t-channel (left).

$$f_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}^{\text{ges}} = f_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}^{\text{ges } L} + f_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}^{\text{ges } T} \quad (5)$$

$$f_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}^{\text{ges } L} = \frac{e^2}{q^2} + f_{\mathbf{p}\mathbf{p}'}^L + g_{\mathbf{p}\mathbf{p}'}^L \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + \frac{j_{\mathbf{p}\mathbf{p}'}^L}{p_f^4} S_{12}(\mathbf{p} \times \mathbf{p}') + \frac{k_{\mathbf{p}\mathbf{p}'}^L}{p_f^2} (\boldsymbol{\sigma} \times \boldsymbol{\sigma}') \cdot (\mathbf{p} \times \mathbf{p}')$$

$$f_{\mathbf{p}\sigma, \mathbf{p}'\sigma'}^{\text{ges } T} = \frac{e^2}{q^2 + i\epsilon} \frac{[\mathbf{p} \cdot \mathbf{p}']^{\perp \mathbf{q}}}{\mu^2} + f_{\mathbf{p}\mathbf{p}'}^T + g_{\mathbf{p}\mathbf{p}'}^T \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + \frac{h_{\mathbf{p}\mathbf{p}'}^T}{p_f^2} S_{12}(\mathbf{q}') + \frac{k_{\mathbf{p}\mathbf{p}'}^T}{p_f^2} (\boldsymbol{\sigma} \times \boldsymbol{\sigma}') \cdot (\mathbf{p} \times \mathbf{p}')$$

where $S_{12}(\mathbf{p}) = [\boldsymbol{\sigma} \cdot \mathbf{p}] [\boldsymbol{\sigma}' \cdot \mathbf{p}] - \frac{\mathbf{p}^2}{3} \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}'$, $\mathbf{q}' = \mathbf{p} - \mathbf{p}'$ and

$$f_{\mathbf{p}\mathbf{p}'}^{L/T}, g_{\mathbf{p}\mathbf{p}'}^{L/T}, h_{\mathbf{p}\mathbf{p}'}^T, j_{\mathbf{p}\mathbf{p}'}^L, k_{\mathbf{p}\mathbf{p}'}^{L/T} \propto \frac{1}{q'^2 - \Pi_{L/T}(0, \mathbf{q}')}.$$

The Landau parameter F_1 is related to the effective mass [4]. To leading order in T one finds

$$\frac{m^*}{\mu} \equiv 1 + \frac{F_1}{3} = -\frac{e^2 v_f}{12\pi^2} \ln \left[\frac{e^2 T}{v_f \mu} \right]. \quad (6)$$

Thus, the leading terms of the effective mass and the Z -factor are identical as they should be. The dynamical screening at finite temperatures regularizes the singularities in the self energy and the effective interaction in a consistent manner. In quark matter the temperature, above which the singularity is effectively washed out, is $\simeq 10^{-4} \mu$, while for metals it is so small that the singularity can safely be neglected [5].

References

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