Analytical expression for cranking inertia of a spheroidal harmonic oscillator

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The hybrid macroscopic-microscopic method can be successfully applied to study nuclear fission phenomena by adding, in a statical approach, to the liquid drop model deformation energy, E_{LDM}, a small shell plus pairing correction, $\delta E = \delta U + \delta P$, which is obtained from a microscopic model by using the Strutinski's procedure [1]. In order to describe the dynamics one needs to know the tensor of inertial coefficients, B_{ij}, which can be computed either phenomenologically (within irrotational hydrodynamics [2] or Werner-Wheeler approximation [3]) or microscopically — with the Inglis's cranking model [1, 4].

In a realistic two center shell model the cranking inertia have to be computed numerically. As an intermediate stage, allowing to test complex computer codes one should develop, we present a simplified case of a spheroidal harmonic oscillator (the simplest Nilsson model) without spin-orbit interaction, for which an analytical result may be obtained. There is one deformation parameter expressed as $\varepsilon = 3(c-a)/(2c+a)$ in terms of semiaxes a, c.

The eigenvalues [5], in units of $\hbar\omega_0^0 = 41A^{-1/3}$ MeV, and the eigenfunctions [4] of the spheroidal harmonic oscillator are given by:

$$\epsilon_i = [N+3/2+\varepsilon(n_\perp - 2N/3)][1-\varepsilon^2(1/3+2\varepsilon/27)]^{-1/3} \quad (1)$$

$$|n_r m n_z\rangle = \frac{\sqrt{2}}{\alpha_\perp}\psi_{n_r}^m(\eta)\frac{1}{\sqrt{2\pi}}e^{im\varphi}\frac{1}{\sqrt{\alpha_z}}\psi_{n_z}(\xi) \quad (2)$$

where the quantum numbers n_z, $n_r = 0, 1, 2, ...n_\perp$, $m = n_\perp - 2n_r$, $N = n_\perp + n_z$. The wave functions $\psi_{n_r}^m(\eta)$ and $\psi_{n_z}(\xi)$ are expressed in terms of associated Laguerre polynomials and Hermite polynomials, respectively. The undimensional variables are defined by: $\eta = \rho^2/\alpha_\perp^2$ and $\xi = z/\alpha_z$ with $\alpha_\perp = \sqrt{\hbar/(M\omega_\perp)}$ and $\alpha_z = \sqrt{\hbar/(M\omega_z)}$.

On the other hand the potential $V(\eta, \xi; \varepsilon) = (\hbar\omega_\perp\eta + \hbar\omega_z\xi^2)/2$ derivative is given by

$$\frac{1}{\hbar\omega_0^0}\frac{dV}{d\varepsilon} = \frac{3}{2}\left[f_1(\varepsilon)\eta + f_2(\varepsilon)\xi^2\right] \quad (3)$$

where

$$f_1 = \frac{\varepsilon(\varepsilon+6)+9}{[27-\varepsilon^2(9+2\varepsilon)]^{4/3}};\ f_2 = 2\frac{\varepsilon(2\varepsilon+3)-9}{[27-\varepsilon^2(9+2\varepsilon)]^{4/3}} \quad (4)$$

For a single deformation parameter the inertia tensor becomes a scalar

$$B_\varepsilon = 2\hbar^2\sum_{\nu\mu}\frac{\langle\nu|\partial V/\partial\varepsilon|\mu\rangle\langle\mu|\partial V/\partial\varepsilon|\nu\rangle}{(E_\nu + E_\mu)^3}(u_\nu v_\mu + u_\mu v_\nu)^2 \quad (5)$$

where E_ν, v_ν, u_ν are the BCS quasiparticle energies and occupation probabilities for quasiparticles and holes, and the summation is performed over the whole number of states ν, μ around the Fermi level, which are considered in pairing interactions. The contribution of the neutron level scheme is added to that of proton levels.

Finally, in order to obtain B_ε in units of $\hbar^2$/MeV, one has to add $b_\varepsilon = b_{\varepsilon 1} + b_{\varepsilon 2} + b_{\varepsilon 3}$ by multiplying each term with $\delta_{n_r' n_r}\delta_{m'm}9/4$ and the result should be divided by $\hbar\omega_0^0$.

$$b_{\varepsilon 1} = \sum_{\nu=k_i}^{k_f}[f_1(2n_r + |m| + 1) + f_2(n_z + 1/2)]^2\frac{(u_\nu v_\nu)^2}{E_\nu^3} \quad (6)$$

$$b_{\varepsilon 2} = \frac{f_2^2}{2}\sum_{\nu\neq\mu}(n_z+1)(n_z+2)\frac{(u_\nu v_\mu + u_\mu v_\nu)^2}{(E_\nu + E_\mu)^3}\delta_{n_z' n_z + 2} \quad (7)$$

$$b_{\varepsilon 3} = \frac{f_2^2}{2}\sum_{\nu\neq\mu}(n_z - 1)n_z\frac{(u_\nu v_\mu + u_\mu v_\nu)^2}{(E_\nu + E_\mu)^3}\delta_{n_z' n_z - 2} \quad (8)$$

Results for ^{240}Pu are presented in figure 1.

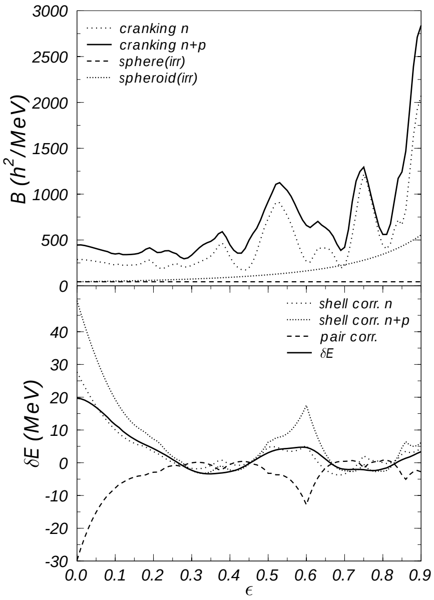

Figure 1: Top: comparison of the effective mass (in units of $\hbar^2$/MeV) calculated microscopically for the proton and neutron level scheme, only for neutrons, the irrotational value and that of a spherical shape. Bottom: shell and pairing corrections. Nucleus ^{240}Pu.

References

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