Thermal Boson Expansion to Next-To-Leading Order

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Spontaneously broken chiral symmetry and its restoration at finite temperature and baryon density is one of the key subjects in strong interaction physics. This problem, often addressed within effective theories with QCD symmetries, requires non-perturbative approaches. In this regard, boson expansion techniques that are frequently used in nuclear and condensed matter systems, are particularly promising. Extensions to provide a consistent thermal boson expansion (TBE) has been discussed in the past [1, 2] but some open issues remained. In a series of papers we have revisited this problem and have proposed an optimal procedure for preserving both the symmetry and the correct quantum statistics [3, 4, 5]. This can be achieved via an extended Holstein-Primakoff (HP) mapping [3] which, in conjunction with the Thermo-Field-Dynamics (TFD) of Takahashi and Umezawa, allows the computation of the leading-order thermodynamics [4]. The method avoids certain conceptual difficulties encountered in Refs. [1, 2].

In one of these papers the starting point is the bosonization of the original fermionic degrees of freedom. Subsequently, the thermalization is achieved by doubling the newly introduced bosonic degrees of freedom as required by the TFD. In such a scenario all thermal quantities are of bosonic type although the original system has Fermi-Dirac statistics. A second possibility, persued in Ref. [1], is to thermalize the original fermions and then bosonize the thermal system by using a realization of some higher algebra such as $O(5)$, for instance. This procedure indeed solves the statistics problem. However, as pointed out in [4], it requires the use of a Hartree-Fock-Bogoliubov (HFB) basis for the thermalized single-fermion states. In the sense of the 1/N-expansion, the HFB approximation is an order mixing approximation and hence violates the symmetries of the problem at hand. To remedy this deficiency we have proposed a new symmetry-conserving scheme [4] which has been successfully applied in Ref. [5] to the evaluation of thermal excitations up to next-to-leading order in the TBE. This scheme requires that the thermal vacuum is a two-mode squeezed state for quasi-fermion operators a^+_{ip} which are the images of the original fermion operators c^+_{ip} via the extended HP mapping. A broken symmetry phase implies that the thermal ground state $|0(T)\rangle$ is a coherent state containing a condensate for the boson operators B as well as their tilde conjugate:

$$|0(T)\rangle = \exp\left[\frac{y_i}{x_i} a^+_{ip}\widetilde{a}^+_{ip} + d\left(B^+ + \widetilde{B}^+\right)\right] |0\rangle_a|\widetilde{0}\rangle_a|0\rangle_B|\widetilde{0}\rangle_B \ ,$$

where $a|0\rangle_a = B|0\rangle_B = 0$ and $\widetilde{a}|\widetilde{0}\rangle_a = \widetilde{B}|\widetilde{0}\rangle_B = 0$, (summation over repeated indices is implied). It should be reiterated that the thermal ground state is not a two-mode squeezed state for the B and $\tilde{B}$ bosons. This is the major difference to the approaches in Refs. [1, 2]. The bosonic modes, introduced via the TBE, are treated as auxiliary modes that organize the Hamiltonian (and other operators) into an expansion in N:

$$H = NH_0 + N^{1/2}H_1 + N^0 H_2 + N^{-1/2}H_3 + N^{-1}H_4 + \dots$$

The indices represent powers of boson and fermion operators, B and a_{ip} and their tilde conjugates. The objective is to diagonalize the thermal Hamiltonian order by order. This amounts to eliminating in the expansion the terms with odd powers in the operators and rearranging those with even powers. In leading order, the relevant contributions H_0, H_1 and H_2 are diagonalized by using techniques familiar from many-body theory namely the BCS and RPA method. The next-to-leading order, on the other hand, calls for a perturbative diagonalization of the remaining parts of the Hamiltonian expressed in the RPA basis. This is accomplished by means of a unitary transformation which removes the off-diagonal terms order by order, in the same spirit as the classical Birkhoff-Gustavson approach [6]. As a consequence, the Hamiltonian is expanded as

$$\begin{aligned} H &= e^{-iS_d/\sqrt{N}} H_d e^{iS_d/\sqrt{N}} = H_d \ + \\ &\frac{1}{\sqrt{N}}[H_d, iS_d] + \frac{1}{2N}[[H_d, iS_d], iS_d] + \dots \ , \end{aligned}$$

where the operator S_d is a function of the fermion and boson operators a_{ip} and B as well as their tilde conjugates. S_d is chosen such that it renders H_4 in diagonal form while eliminating H_3 at the same time. The approach, sketched above, has been applied to a simple model system that can be solved analytically. The expansion yields convergence of the thermal boson expansion [5] towards the exact solution. An important application will be the vacuum and in-medium hadron scattering in next-to-leading order. This is simply given by the tree level approximation with the renormalized fields and couplings of H_4.

References

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