Symmetry properties of self-consistent Schwinger-Dyson equations[G]

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The description of strongly interacting systems like dense and hot hadronic or QCD matter requires non-perturbative approximation schemes which resum certain sets of diagrams. A class of such approaches proposed in the early 60ies within non-relativistic many-body theory is given by the so-called Φ-derivable approximations[1, 2, 3], later extended to relativistic quantum field theory within the path-integral formalism[4].

These self-consistent approximations are constructed through a generating functional $\Gamma[\varphi, G]$, depending on the mean fields φ and the dressed propagators G where the interaction part is specified by the Φ-functional. The latter is given by two-particle irreducible closed diagrams with point-vertices defined as in perturbation theory and lines standing for dressed rather than free propagators. Truncated to a set of practically feasible diagrams it defines a Φ-derivable approximation. The equations of motion themselves are determined by the stationary points of the so contructed approximate Γ under simultaneous variations with respect to φ and G, this way defining the driving sources for the approximate *mean-field equation* of motion and the *Dyson-equation*, where the self-energy is defined by $\Sigma = 2\mathrm{i}\delta\Phi/\delta G$.

The most important features of such approximation schemes are that the expectation values for Noether-currents pertaining to global symmetries of the classical action are exactly conserved and that in the case of thermal equilibrium a consistent calculation of thermodynamic variables (e.g., pressure or entropy) is guaranteed.

As a further important and till recently unknown feature of such approximation schemes is their renormalizability in the usual sense whenever the underlying theory is perturbatively renormalizable. The proof has been given recently by us[5]. Also first numerical investigations beyond the leading mean-field order giving rise to the self-consistent treatment of particles with finite mass width could be established for the ϕ^4-model [6].

However one of the most prominent problems with Φ-derivable approximations is the violation of Ward-Takahashi identities of symmetries for the two-point and higher n-point functions. Especially for spontaneously broken theories the Goldstone modes do not appear to be massless within the approximation as they should [7].

In the present work [8] we have proven on the example of the O(N)-model that the origin of this problem lies in a *violation of crossing symmetry* at orders of the expansion parameter (which may be the coupling constant, number of loops ($\hbar$) or the $1/N$) beyond that taken explicitly into account for the definition of the approximate Φ-functional. Solving the coupled set of self-consistent equations of motion implies a resummation of the self-energy only in certain channels (s channel) to any order of the expansion parameter while contributions from other channels (t and u channels) are missing at orders of the expansion parameter higher than that taken into account in the approximate Φ-functional.

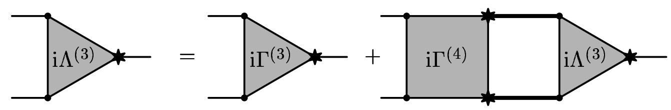
It was shown that for any such Φ-derivable approximation one can repair the symmetries by defining a non-perturbative approximation through a new effective action by

$$\tilde{\Gamma}[\varphi] = \Gamma[\varphi, \tilde{G}[\varphi]]. \tag{1}$$

Here $\Gamma[\varphi, G]$ is the generating functional of the considered Φ-derivable approximation, while $\tilde{G}[\varphi]$ is the propagator defined as the stationary point of Γ with an arbitrarily given mean field φ. The method provides that minimal set of extra diagrams to those given by the considered Φ-derivable approximation which is necessary to repair the symmetry. It can be renormalized along the same line of arguments as given for the self-consistent approximations. Since $\tilde{\Gamma}$ is invariant under the symmetry, local symmetric counterterms, which are independent of temperature, are sufficient to render $\tilde{\Gamma}$ finite.

It turns out that the approximations for the proper vertex functions $\Gamma^{(n)} = \delta^n\tilde{\Gamma}/\delta\varphi^n$ are not only crossing symmetric by construction but also fulfill the usual Ward-Takahashi identities of the underlying symmetry.

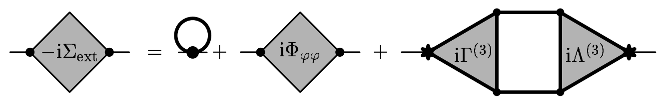
The extra diagrams for the self-energy are obtained through a Bethe-Salpeter (BS) equation for the three-point function, $\Lambda^{(3)}$, diagrammatically given as follows



Both, its inhomogeneity and kernel have to comply with the chosen approximate Φ-functional:

$$\begin{aligned}
\Gamma^{(3)}_{1j,2k;3l} &= \left[\frac{\delta^3 S[\varphi]}{\delta\varphi_1^j\delta\varphi_2^k\delta\varphi_3^l} - 2\mathrm{i}\frac{\delta^2\Phi[\varphi,G]}{\delta G_{12}^{jk}\delta\varphi_3^l}\right]_{G=\tilde{G}[\tilde{\varphi}],\ \varphi=\tilde{\varphi}} \\
\Gamma^{(4)}_{1j,2k;3l,4m} &= -2\left[\frac{\delta^2\Phi[\varphi,G]}{\delta G_{12}^{jk}\delta G_{34}^{lm}}\right]_{G=\tilde{G}[\tilde{\varphi}],\ \varphi=\tilde{\varphi}},
\end{aligned}$$

where $S[\varphi]$ is the corresponding classical action. The solution of the BS-equation permits to construct a so called external self-energy (on top of the self-consistent Φ-derivable approximation) which preserves the symmetries



Exploratory cases for the chiral linear σ-model (O(4)) are discussed and calculated, where the appearance of proper Nambu-Goldstone modes in case of spontaneously broken chiral symmetry in the external self-energy could be shown.

The results are important for the construction of non-perturbative effective models beyond the usually considered mean field level for strongly interacting systems, as in QCD, or dense hadronic matter, where renormalization questions and symmetry considerations are vital ingredients of the theory.

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