Asymmetric saddle-point shapes from an integro-differential equation

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The equilibrium nuclear shapes in fission theory are usually obtained by minimizing the deformation energy for a given surface equation [1]. We present a method [2] allowing to find a saddle-point shape as a solution of an integro-differential equation; no *apriori* surface equation has to be given. In the approach based on a pure liquid drop model (LDM) [3], saddle-point shapes are always reflection symmetric: the deformation energy increases with the mass-asymmetry parameter $\eta = (A_1 - A_2)/(A_1 + A_2)$, as is illustrated in Fig. 1 (top), where η is replaced by an almost linear dependent quantity $(d_L - d_R)/R_0$. In this way the well known fission fragment mass asymmetry can not be explained. By adding the shell corrections δE, $E_{def} = E_{LDM} + \delta E$, we can obtain the minima shown in Fig. 1 (bottom) and Fig. 2. The surface equation of

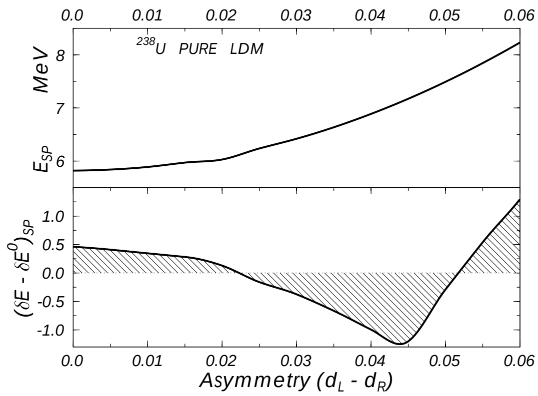

Figure 1: Saddle-point deformation energy versus mass asymmetry parameter for the binary fission of ^{238}U within a pure LDM (top) and the contributionof shell corrections (bottom).

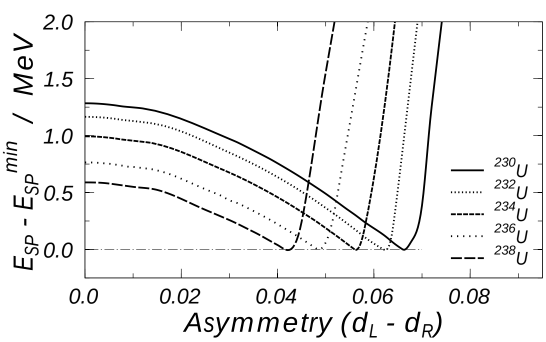

Figure 2: Saddle-point deformation energy versus mass asymmetry parameter for the binary fission of some U isotopes in the presence of shell corrections.

an axially symmetric nucleus, $u(x)$, is a solution of the following integro-differential equation:

$$u'' = 2 + \frac{1}{u}[u'^2 + (x - d + V_s)(4u + u'^2)^{3/2}] \qquad (1)$$

where d is an input parameter (d_L and d_R for the left-hand side and right-hand side of the shape, respectively) which determines the deformation and V_s is a Coulomb potential, expressed by a surface integral. In our approach we included in the deformation energy

$$E(R) = E_{LD}(R) + \delta E(R) - \delta E^0 \qquad (2)$$

a phenomenological shell corrections δE inspired from [4], and the differential equation is solved iteratively by using Runge-Kutta method. The procedure is repeated until the solution leads to the minimum of the deformation energy (surface plus Coulomb plus shell corrections).

At a given deformation we find the fragment volumes and the corresponding number of protons and neutrons $Z_i(R)$, $N_i(R)$ $(i = 1, 2)$. For every fragment we add contributions from protons and neutrons

$$\delta E(R) = \sum_i \delta E_i(R) = \sum_i [\delta E_{pi}(R) + \delta E_{ni}(R)] \qquad (3)$$

given by

$$\delta E_{pi} = Cs(Z_i); \quad \delta E_{ni} = Cs(N_i) \qquad (4)$$

where

$$s(Z) = Z^{-2/3}F(Z) - cZ^{1/3} \qquad (5)$$

and a similar equation for $s(N)$, where $F(n)$ is an algebraic expression which becomes minimum when a proton or neutron number reaches a magic value. The parameters $c = 0.2$, $C = 6.2$ MeV were determined by fit with experimental masses and deformations. By introducing shell

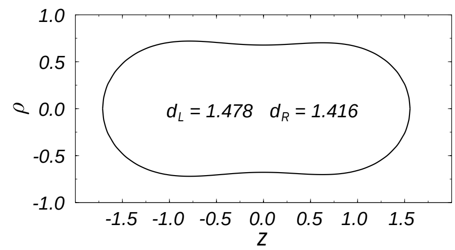

Figure 3: Mass-asymmetric saddle-point shape of ^{232}U obtained as a solution of integro-differential equation in the presence of shell corrections.

corrections we obtained minima of deformation energy illustrated in Fig. 2 for $^{230-238}$U isotopes at a finite mass asymmetry. The corresponding shape is plotted in Fig. 3.

References

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