The Criteria for the Existence of Coulomb Strings in Storage Rings

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We derive four approximate conditions for the stability of Coulomb chains in circular storage rings. These criteria are well met by the existing data from experiments in SIS, ESR and Cryring.

In the recent years Schottky measurements of the momentum spread $\delta p/p$ at beams with very low density and extremely electron cooled heavy ions in the storage ring ESR [1] and in the synchroton SIS [2] of GSI have revealed that below a certain threshold in density intrabeam scattering ceases to act. Similar observations have been made in the Cryring [3] of Stockholm. The beam radius below threshold was determined to be smaller than some ten micrometers.

The typical jump of the momentum spread has been explained as the ions form a linear string where the particles cannot pass each other any more [4]. Hereby classical Monte-Carlo trajectory calculations were performed with charged particles heading towards each other under constant focusing with the betatron frequency ω_β of the respective ring. As results, the probability of reflection (or transmission) drops (increases) sharply when going to larger interparticle distances.

Although this model can explain the given experimental data it cannot yet predict the density and longitudinal, $T_{||}$, and transverse, $T_\perp$, thermal energies at which this effect shows up. The only hitherto known necessary relation for the existence -but not the stability- of Coulomb strings,

$$d > 1.4\ a_{\rm WS}, \tag{1}$$

stems from the transition from strings to zigzags at $\lambda < 0.709$, where $\lambda = a_{\rm WS}/d$ is the linear density and $a_{\rm WS}$ is the Wigner-Seitz radius $a_{\rm WS}^3 = 3q^2/2M\omega_\beta^2$ with q being the charge and M the mass of the ions.

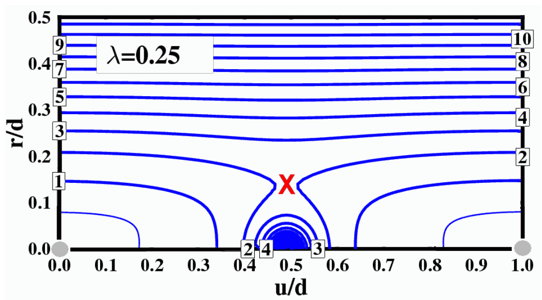

Figure 1: Potential energy of the zigzag motion. The numbers in boxes label the contour lines in units of q^2/d. The ground state is indicated by the light dot and the saddle point is marked by a cross.

Suppose that at rest the ions are distributed equidistantly along the z-axis, and that this string can vibrate in a zigzag fashion transversally as well as longitudinally by moving towards and away from each other. A contour plot of this potential is shown in Fig. 1 vs. the longitudinal displacement u of two adjacent ions and the radial excursion r. Upwards, the system needs energy by working against the radial harmonic confining potential. Going to the right means that the ions approach each other with $u = d/2$ being at closest contact. There, without radial displacement the potential exhibits an infinitely high mountain. For an intermediate radial amplitude, however, the potential shows a saddle point at close contact which can only be overcome if the longitudinal kinetic energy is sufficiently high. Note that the height of the saddle point vanishes by definition at the critical $\lambda = 0.709$ where the ground state string turns into a zigzag. It can be shown that the radial position in units of $a_{\rm WS}$ is almost constant in a wide range of density if λ becomes small. The same applies to its height in units of $q^2/a_{\rm WS}$. As a result we derive the following conditions for the stability of Coulomb strings in circular rings:

$$r \gtrsim 0.55\ a_{\rm WS} \quad \text{or} \quad T_\perp \gtrsim 0.25\ q^2/a_{\rm WS} \tag{2}$$
$$T_{||} \gtrsim 0.7\ q^2/a_{\rm WS} \tag{3}$$

Hereby the second of eq. (2) was obtained from $T_\perp = M\omega_\beta^2 r^2/2$ and the longitudinal kinetic energy can be converted to momentum spread with help of the relation $T_{||} = M(\beta c\ \delta p/p)^2/(8/\ln 2)$ [1], where βc is the beam velocity. Note that the relations (2,3) contain as units of distance just the Wigner-Seitz radius and not the interparticle distance. This reflects the fact that most of the time the ions do not interact notably. Only if they come close within a distance of the order of $a_{\rm WS}$ they feel the properties of the potential.

The last relation is derived from the stiffness of the potential of Fig. 1 at the ground state $(r = 0, u = 0)$ along the u-axis marked by the light dot. Equating the saddle energy to $M\omega_{||}^2/2$, one obtains the Coulomb period $\tau_{||} = 2\pi/\omega_{||}$. Thus the ratio of Coulomb period to betatron period, hence, must be

$$\tau_{||}/\tau_\beta \gtrsim \lambda^{-1}, \tag{4}$$

reflecting the fact that the period of an average Coulomb scattering must be large as compared to a single betatron period. This condition is well fulfilled for the experiments.

The idea of this work originated after a seminar at GSI of Dieter Möhl who reported on calculations of the luminosity of an electron-ion collider with an ordered ion beam at RIKEN [6].

References

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