Finite-Temperature Gluon Condensate with Renormalization Group Flow Equations

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QCD at low energies can be approximated by effective models which have to satisfy the observed symmetry properties and anomaly structure of the theory. Besides the important approximate and spontaneously broken chiral symmetry at low energy, the classical QCD-Lagrangian exhibits an additional scale invariance in the limit of vanishing current quark masses which is embedded in a larger conformal group. This symmetry is broken at the quantum level by radiative corrections – the so-called trace anomaly. In order to mimic the trace anomaly in an effective framework one introduces a scalar ($J^{PC} = 0^{++}$) dilaton field χ with scaling dimension one and a logarithmic potential with coupling h of the form [1]

$$V = h\left(\frac{\chi}{\chi_0}\right)^4 \ln\left(\frac{\chi}{\chi_0 e^{1/4}}\right) . \qquad (1)$$

This potential breaks scale invariance and leads therefore to a finite vacuum expectation value (VEV) $\chi_0 \equiv \langle 0|\chi|0\rangle$. This effective realization allows for an identification of the gluon condensate $\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle$ with the VEV χ_0 of the effective theory with broken scale invariance. Here $G_{\mu\nu}$ denotes the non-Abelian field strength tensor of QCD. Following the suggestion of Campbell et al. [2] χ_0 can be regarded as an effective order parameter for the deconfining phase transition.

In order to investigate the possible gluonic phase transition we perform a finite-temperature analysis of the effective theory within the framework of a proper-time renormalization group (PTRG) approach [3]. Within the PTRG approach we can directly calculate the full non-truncated effective dilaton potential for any value of the field and not just at the minimum, χ_0. This in principle allows to analyze the order of the phase transition.

In this work the glueball mass m_χ and bag constant B are taken as an input. We choose values around $m_\chi = 1.5$ GeV which is motivated by recent lattice results and vary the bag constant in between 0.149^4 and 0.276^4 GeV^4. From previous studies it is observed that the temperature scale, where thermal excitations become important, is determined by the value of the bag constant at $T = 0$. For $B^{1/4} = 0.14$ GeV, the onset of a significant shift of the minimum is seen at $T \sim 0.25$ GeV while for $B^{1/4} = 0.24$ GeV this happens only for temperatures above $T \sim 0.4$ GeV. The value for the "critical" temperature T_c is dominated by the value of the bag constant. A larger bag constant results in a higher critical temperature.

In Fig. 1 the change of the normalized 'order parameter' $\chi_0^4(T)/\chi_0^4(0)$ with temperature is shown for two different chosen bag constants B but equal glueball masses $m_\chi = 1.5$ GeV. For both bag constants the condensate stays almost constant up to temperatures of the order 200 MeV which is beyond the chiral phase transition temperature for two quark flavors.

In Fig. 2 the temperature-dependent glueball mass m_χ is displayed for two different bag constants. The mass as function of temperature is independent on the bag constant below temperature of 180 MeV. For the larger bag constant $B^{1/4} = 0.276$ GeV it then decreases and increases again for very large temperatures above 800 MeV. For the smaller bag constant $B^{1/4} = 0.149$ GeV the mass grows almost linearly with temperature around $T \sim 350$ MeV. At very high temperatures such a behavior is expected since perturbation theory is applicable.

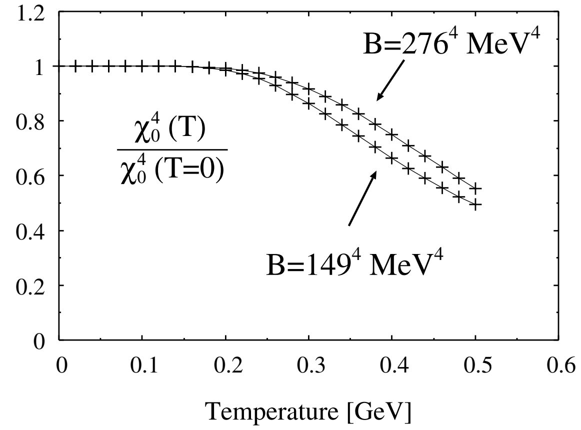

Figure 1: The temperature behavior of the normalized gluon condensate $\chi_0^4(T)/\chi_0^4(0)$ for two different bag constants.

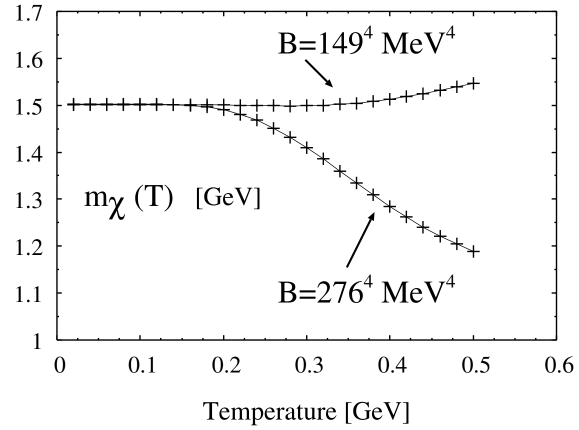

Figure 2: The glueball mass versus temperature for two different bag constants.

References

[1] J. Schechter, Phys. Rev. **D21** (1980) 3393;

[2] B.A. Campbell, J. Ellis and K.A. Olive, Phys. Lett. **235B** (1990) 325; Nucl. Phys. **B345** (1990) 57.

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