Boson Expansion For Quantum Field Theory

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For the last decades Boson expansion theory has played an effective role in the development of approximate solutions to the many-body problem in nuclear physics [1]. These techniques also provide promising tools to address solutions for quantum field theories [2]. In a recent work, we have succeeded in combining an extended form of the Holstein-Primakoff mapping (EHPM) [3] with thermo-field-dynamics [4] to generate a consistent thermal boson expansion for quantum mechanical systems such as the bosonic $O(N)$ anharmonic oscillator as well as the fermionic Lipkin model [7]. The analogue of the first of these two models in four space-time dimensions can be used to address important questions such as the chiral transition for two flavors QCD or the scattering of pions at zero and finite temperature. It is also found that the EHPM removes number of ambiguities when computing anharmonicities in the next-to-leading order of the $1/N$ expansion [7], which are very much relevant to the $\pi\pi$ scattering problem, for instance. Furthermore, the EHPM allows to extend the use of the boson expansion approach to quantum systems with internal $SU(N)$ symmetry where one is confronted with theories having tensor couplings

$$\mathcal{L}_I = f_{ijkl}\phi_i\phi_j\phi_k\phi_l + g_{ijk}\phi_i\phi_j\phi_k + ... \ , \qquad (1)$$

and which calls for a simultaneous bosonization of both pair and single modes. This opens, indeed, the possibility to ultimately address the question of the chiral transition in $SU(3)$ flavor QCD in four space-time dimensions. This point, although crucial for the understanding of the QCD phase diagram, has not been adequately addressed before due to the fact that the $1/N_f$ expansion cannot be correctly sorted out with known techniques [6].

The approach highlighted above and which fundamentally relies on the bosonization of the generators of the $Sp_2 \otimes N_1$ semidirect product group,

$$\begin{aligned} (aa)_I &= \sqrt{2+4(n+m)}A \ , \qquad (a^+a)_I = 2n+m \ , \\ (a)_I &= \sqrt{2+4(n+m)}\,\Gamma(m)\alpha + 2\alpha^+ A\,\Gamma(m) \ , \\ (a^+a^+)_I &= (aa)_I^+ \ , \qquad (a^+)_I = (a)_I^+ \ , \\ \Gamma(m) &= \left[\frac{m-1}{2(2m+1)(2m-1)}\right]^{\frac{1}{2}} , \\ n &= A^+A \ , \qquad m = \alpha^+\alpha \ , \end{aligned} \qquad (2)$$

is very appealing from many aspects and particularly from its symmetry conserving character. However, it has a serious drawback. The (asymptotic) particles constructed in that way are 'Hartree' particles, thus semi-classical objects. They are also the building blocks for all higher n-point functions in the theory. To assume the full wave function of the quantum particles, various attempts were made in the context of the Gaussian functional approach (GFA) [8] but failed, so far, to correct for this problem. Indeed, assuming the full wave function of the quantum states induces an uncontrollable order mixing which inevitably destroys the symmetries.

In the case of chiral symmetry, a method that allows to build a non-perturbative pion state was suggested in [2]. This consists in mapping the canonical pion field into the axial current. The idea is supported by the exact (Goldstone) statement: $Q_5^a|vac\rangle \propto |\pi^a\rangle$, which allows to build a pion state by acting with the symmetry generator on the correlated vacuum of the theory. In an effective model, the generator Q_5^a is simply given by Noether's theorem. Therefore one can use the field structure of Q_5^a to model an excitation operator for the asymptotic pion field. This point was successfully developed in several papers and the approach was shown to accommodate finite temperature and chemical potential situations.

In spite of the fact that this technique gives the first approximate but fully quantum solution to the problem, it does not provide, however, a systematic procedure which helps one to tackle the anharmonicities beyond the random phase approximation (RPA). Here again the answer to this question can be found in the boson expansion theory. However, since in most cases (except for gauge symmetry) the symmetry generator is a one-body operator, one has to create a new mapping which can accommodate the bosonization of pairs of two different kinds of bosons at least. Technically this amounts to a new realization of the algebra of the symplectic Sp_4 group which was already derived in [3]. However, in the case of broken symmetries one needs to take into account single boson mapping as well. Thus one needs to make a realization of the algebra of the 15 generators-large semidirect product group $Sp_4 \otimes N_2$. As pointed out above, since this new bosonization is aimed at preserving the quantum statistic of the particles one expects to finally obtain, in the case of the $\pi\pi$ scattering, the first non-perturbative solution with the crossing symmetry built in. This important symmetry is simply lost in the $1/N$ solution.

References

[1] A. Klein, E. R. Marshalek, Rev. Mod. Phys. 63 (1991) 375.
[2] Z. Aouissat, P. Schuck, J. Wambach, Nucl. Phys. A618 (1997) 402
[3] Z. Aouissat, Phys. Rev. C62 (2000) 012201(R).
[4] Y. Takahasi, H. Umezawa, Coll. Pheno., 2 (1975) 55.
[5] Z. Aouissat, A. Storozhenko, A. Vdovin, J. Wambach, Phys. Rev. C64 (2001) 015201
[6] H. Meyer-Ortmanns, B-J. Schaefer, Phys. Rev. D53 (1996) 6586.
[7] Z. Aouissat, A. Storozhenko, A. Vdovin, J. Wambach, work in progress.
[8] G. A. Camelia, Phys. Lett. B407 (1997) 268.
[9] Z. Aouissat, J. Wambach, work in progress.