Remarks on Delta Radiative and Dalitz Decays

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The $\Delta(1232)$ resonance is expected to give an important contribution to the dilepton yield in nucleon-nucleon and heavy-ion collisions. In refs. [1, 2, 3, 4, 5], expressions are derived for the $\Delta \to Ne^+e^-$ decay rate, which, however, are not equivalent with respect to the kinematical factors. In refs. [2, 5, 6, 7], the radiative decay $\Delta \to N\gamma$ is calculated. The results, surprisingly, are also not equivalent. We thus give an independent calculation of these two decays.

In terms of helicity amplitudes, the decay width of a resonance, R, decaying into a nucleon, N, and a photon, γ^*, can be written as

$$\Gamma(R \to N\gamma^*) = \frac{k}{8\pi m_R^2(2J_R+1)} \sum_{\lambda\lambda'\lambda''} \left| < \lambda|S|\lambda'\lambda'' > \right|^2 \tag{1}$$

where m_R is the resonance mass, J_R its spin, k is the photon momentum in the resonance rest frame, λ, λ', and λ'' are the resonance, nucleon, and photon helicities, and $< \lambda|S|\lambda'\lambda'' >$ the corresponding amplitudes.

For the $\Delta \to N\gamma^*$ transition, there are three independent helicity amplitudes which can be found in paper by Jones and Scadron [8], eqs.(18). Using these amplitudes, we obtain the Δ resonance width for decay into a nucleon and a virtual photon:

$$\begin{aligned}\Gamma(\Delta \quad \to \quad N\gamma^*) &= \frac{\alpha}{16}\frac{(m_\Delta+m_N)^2}{m_\Delta^3 m_N^2}((m_\Delta+m_N)^2 - \\ &-M^2)^{1/2}((m_\Delta-m_N)^2-M^2)^{3/2} \\ &\left(G_M^2+3G_E^2+\frac{M^2}{2m_\Delta^2}G_C^2\right).\end{aligned} \tag{2}$$

Here, m_N and m_Δ are the nucleon and Δ masses, $M^2 = q^2$ where $q_\mu = (\omega, 0, 0, k)$ is the photon four-momentum, G_M, G_E, and G_C are magnetic, electric and Coulomb transition form factors, as defined in ref. [8], eqs.(15).

The factorization prescription (see *e.g.* [9]) allows to find the dilepton decay rate of the Δ resonance:

$$d\Gamma(\Delta \to Ne^+e^-) = \Gamma(\Delta \to N\gamma^*) M\Gamma(\gamma^* \to e^+e^-)\frac{dM^2}{\pi M^4}, \tag{3}$$

with

$$M\Gamma(\gamma^* \to e^+e^-) = \frac{\alpha}{3}(M^2+2m_e^2)\sqrt{1-\frac{4m_e^2}{M^2}} \tag{4}$$

being the decay width of a virtual photon into the dilepton pair with the invariant mass M.

The physical $\Delta(1232) \to N\gamma$ decay rate is given by eq.(2) at $M = 0$. The last three equations being combined give the $\Delta(1232) \to Ne^+e^-$ decay rate.

In ref. [1] the $\Delta \to Ne^+e^-$ transition is calculated. The width $\Delta \to N\gamma$ can be extracted from eqs.(10) - (12) of this work. It does not coincide with our eq.(2). The $M = 0$ limits of eqs.(4.9) - (4.13) of ref. [2] and of eqs.(3) - (13) of ref. [5] do not coincide with our eq.(2) also. In ref.[6], the physical $\Delta \to N\gamma$ decay is calculated in the light cone QCD assuming $F_1 = \sqrt{\frac{3}{2}} g_{\Delta N\gamma} \neq 0$ and $F_2 = F_3 = 0$, with the form factors F_i defined as in ref. [8], eq.(4), and the coupling constant $g_{\Delta N\gamma}$ defined as in ref.[6], eq.(3). Using eqs.(15) of ref. [8] and our eq.(2), we obtain an expression for the $\Delta \to N\gamma$ width, which differs from eq.(13) of ref.[6] (by a factor of 2/3 in the heavy-baryon limit). In ref. [7], an expression is derived for the radiative decay of a spin J_R baryon resonance. We agree with eq.(2.59) of this work. The results [1, 2, 5, 6, 7] for the $\Delta \to N\gamma$ decay are distinct from each other.

Our result for the $\Delta \to Ne^+e^-$ width, eqs.(2) - (4), is distinct from the results of refs. [1, 2, 5], since we disagree already on the $\Delta \to N\gamma$ width. In ref. [3], the $\Delta \to Ne^+e^-$ decay is calculated using the chiral perturbation theory. We reproduce kinematical factors of the $M1$ part of the decay width in eq.(2) of ref. [3]. In the soft dilepton limit, $m_e = 0$ and $M \to 0$, we agree also with the $E2$ part, but disagree with it at finite values of M. Our expression for the Δ decay rate differs from that of ref. [4], eqs.(8) and (9). The results of refs. [1, 2, 3, 4, 5] for the $\Delta \to Ne^+e^-$ decay are distinct from each other.

In the first order of the perturbation, the decay rates of the on-mass-shell spin-3/2 particles are well defined theoretically. The discrepancies between refs.[1-6] and our paper [10] can probably be attributed to errors in calculations of refs.[1-6].

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