Determination of the Equation of State of Dense Matter

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Features of the nuclear equation of state (EOS) at supra-normal densities ρ can be inferred from flow in collisions of heavy nuclei at high beam energies E_{lab}. At low impact parameters, large regions of high ρ are formed and matter is best equilibrated. The collective flow can provide access to pressure generated in the collision, as seen in the hydrodynamic Euler equation for the nuclear fluid (in a frame where the collective velocity vanishes, $v = 0$): $(e+p)\,\frac{\partial}{\partial t}\,\vec{v} = -\vec{\nabla} p$. The velocity becomes an observable at the end of a reaction. In comparing to the Newton equation, we see that the pressure $p = \rho^2 \frac{\partial(e/\rho)}{\partial \rho}$ plays role of a potential for the hydrodynamic motion, while the sum of energy density e and p plays the role of mass. At moderate energies, we have $e + p \approx \rho\, m_N$ and we see that the collective flow can tell us about the pressure in comparison to ρ. For that, we need to know the spatial size where the p-gradients develop and this will be determined by the nuclear size. However, we also need the time during which the hydrodynamic motion develops.

Equilibrium, required for hydrodynamics that gave insights, is not quite achieved in reactions and transport theory is called for. In theory based on the Boltzmann equation, the system is described in terms of the distribution functions f for different particles. Net energy is a functional of f and can be parametrized to yield different EOS. The f follow a set of the Boltzmann equations with 1-particle energies $\epsilon = \delta e/\delta f$: $\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{p}}\,\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon}{\partial \mathbf{r}}\,\frac{\partial f}{\partial \mathbf{p}} = I$, where I is the collision integral.

The first observable to consider for extracting EOS is the net radial collective energy which may reach as much as half of all energy in a reaction. Despite its magnitude, this energy is not useful for EOS because of the lack of information on time Δt during which the expansion develops. Large p acting over a short Δt can produce the same collective energy as low p acting over a long Δt. This makes apparent the need for a timer.

The timer role may be taken on by the spectator matter in the periphery of an energetic reaction, proceeding virtually at original velocity. As the central participant matter expands after compression, the spectators shadow its expansion. If central p are high and the expansion is rapid, the anisotropies generated by the spectators are strong. On the other hand, if p are low and, correspondingly, the expansion of the matter is slow, the shadows left by spectators cannot be very pronounced.

The spectators can produce different anisotropies in participant emission. Early on, the participant matter gets locked in a channel between the spectators, titled at an angle. Hence, the forward and backward emitted particles get an average deflection away from the beam axis. Another anisotropy results for particles with no longitudinal velocity. The central region is open to the vacuum in the direction perpendicular to the reaction plane, but in the direction within the reaction plane the region is shadowed by the spectators. Thus, more particles can be transversally emitted perpendicular to than in the plane direction.

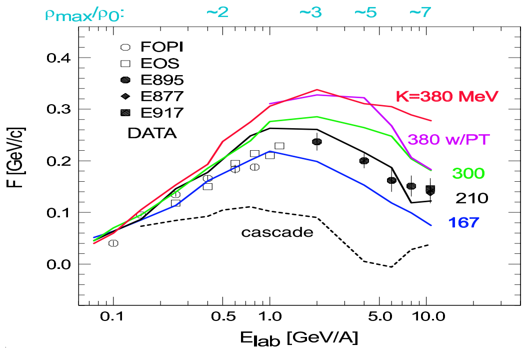

Figure 1: Sideward flow in Au + Au [1].

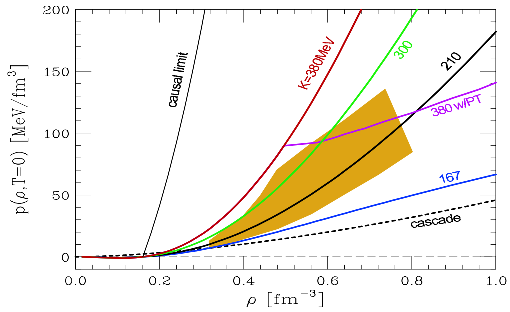

Figure 2: Constraints from flow on the $T = 0$ pressure-density relation, indicated by the shaded region [1].

The different anisotropies have been measured over a wide range of E_{lab}. Fig. 1 shows the measure F of the sideward deflection in Au + Au, vs E_{lab}. Symbols represent data and lines – simulations for different EOS. On top of the figure, maximal ρ reached at each E_{lab} are shown. Without interaction contributions to p, the simulations labelled cascade produce far too weak anisotropies. The simulations with EOS characterized by the incompressibility $K = 167$ MeV yield adequate F at lower E_{lab}, but too low at higher E_{lab}. On the other hand, for the EOS characterized by $K = 380$ MeV, F appears too high at virtually all E_{lab}. Here, K merely label the different utilized EOS, as p in the expansion are generated at much higher ρ than normal ρ_0 and, in fact, changing in the course of the reaction.

No one EOS allows to describe both types of measured anisotropies at all E_{lab}. The $K = 210$ MeV EOS is best for F, and the $K = 300$ MeV EOS is the best for the other elliptic anisotropy. We use the discrepancy as a measure of inaccuaracy of the theory and draw broad boundaries on $p(\rho)$ from what is common in the conclusions. To ensure that the effects of compression dominate in the reaction, we limit ourselves to $\rho > 2\rho_0$. The boundaries on p are shown in Fig. 2 and they eliminate some of the more extreme models for EOS utilized in nuclear physics, such as the NL3 model and models assuming a phase transition at relatively low ρ.

[1] P. Danielewicz *et al.*, submitted to Nature.