# Isoscaling and its Statistical Interpretation.

A.S. Botvina and W. Trautmann, *GSI Darmstadt.*

The scaling properties of cross sections for fragment production with respect to the isotopic composition of the emitting systems were studied long ago in light-ion induced reactions [1]. Recently, the isoscaling was also reported for heavy-ion reactions [2]. It is constituted by the exponential dependence of the production ratios $R_{21}$ for fragments with neutron number $N$ and proton number $Z$ in reactions with different isospin asymmetry:

$$R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} = C \cdot exp(N \cdot \alpha + Z \cdot \beta), \quad (1)$$

with three parameters C, $\alpha$ and $\beta$. Here $Y_2$ and $Y_1$ denote the yields from the more neutron rich and the more neutron poor reaction system, respectively. In some reactions, the parameters $\alpha$ and $\beta$ have the tendency to be quite similar in absolute magnitude but of opposite sign [2]. This suggests an approximate scaling with the third component of the isospin $t_3 = (N-Z)/2$ of the form

$$R_{12}(N,Z) = Y_1(N,Z)/Y_2(N,Z) = C \cdot exp(-t_3 \cdot \beta_{t3}). \quad (2)$$

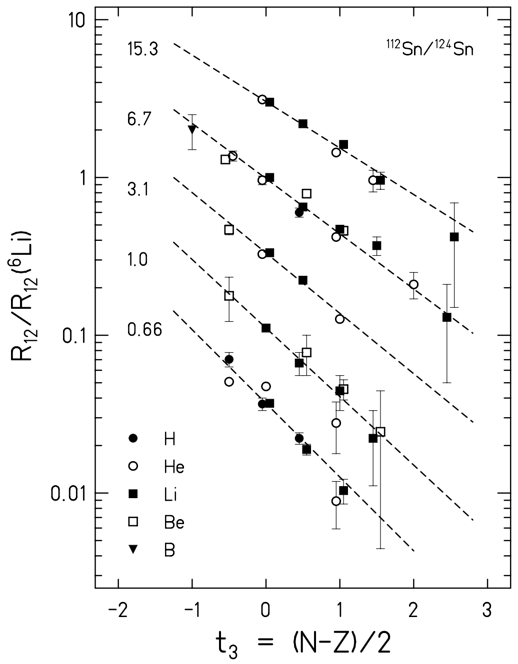


Figure 1: Normalized isotopic effect $R_{12}$ for five reactions, labeled with the projectile energy (in GeV) and shifted by factors of three for clarity. The lines are the results of the exponential fits according to Eq. (2).

The isotope ratios measured in five pairs of reactions induced by light particles from protons of 0.66 GeV to $\alpha$-particles of 15.3 GeV incident energy on $^{112}$Sn and $^{124}$Sn targets, and normalized with respect to the ratio for $^6$Li, are shown in Fig. 1. The parameter $\beta_{t3}$ decreases from 1.08 to 0.68 with increasing energy [3]. The inverse scaling parameter $1/\beta_{t3}$ is found to increase approximately in proportion to the isotope temperature deduced from the yields of He and Li isotopes.

The isoscaling phenomenon arises naturally in a statistical fragmentation mechanism. In the grand canonical approximation of the Statistical Multifragmentation model (SMM) [4] the mean yield of a fragment with mass number $A$ and charge $Z$ is given by

$$\langle Y_{AZ} \rangle = g_{AZ} \frac{V_f}{\lambda_T^3} A^{3/2} \exp\left[-\frac{1}{T}\left(F_{AZ} - \mu A - \nu Z\right)\right], \quad (3)$$

where, within the brackets, $T$ is the temperature, $F_{AZ}(T)$ is the individual free energy of the fragment, and $\mu$ and $\nu$ are the chemical potentials responsible for the mass and charge conservation in the system, respectively. As evident from Eq. (3), for two systems 1 and 2 with different total mass numbers ($A_1$ and $A_2$) and charges ($Z_1$ and $Z_2$) but with the same temperature and density, the ratio of fragment yields produced in these systems is given by Eq. (1) with parameters $\alpha = (\mu_1 - \mu_2)/T$ and $\beta = ((\mu_1 - \mu_2) + (\nu_1 - \nu_2))/T$.

In the SMM the isoscaling coefficients are connected with properties of the produced fragments. In particular, their symmetry energy is $E_{AZ}^{sym} = \gamma(A-2Z)^2/A$, where $\gamma \simeq 25$ MeV is the symmetry energy parameter, reflecting also the symmetry energy of nuclear matter. We have obtained that the potential differences depend essentially only on the coefficient $\gamma$ of the symmetry term and on the isotopic compositions of the sources:

$$\mu_1 - \mu_2 \approx -4\gamma\left(\frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2}\right),$$
$$\nu_1 - \nu_2 \approx 8\gamma\left(\frac{Z_1}{A_1} - \frac{Z_2}{A_2}\right). \quad (4)$$

This formula is relevant for any temperature in the grand-canonics, and it was confirmed for the multifragmentation region ($T \gtrsim 5$ MeV) with the microcanonical Markov chain SMM calculations [3]. We suggest to use the isoscaling phenomenon for experimental determination of the symmetry energy coefficient [3]: 1) First, the isoscaling parameters and the isospin of the thermal sources should be extracted. 2) The temperature should be found in an independent way, e.g. as the isotope temperature. 3) By using Eq. (4) one can find $\gamma$. 4) Since the secondary deexcitation of the produced fragments can influence the isoscaling parameters, the corresponding correction has to be applied.

## References

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