Short-Range Repulsive and Tensor Correlations in Nuclei

T. Neff and H. Feldmeier (GSI)

It is a central challenge of nuclear physics to explain the properties of nuclear many-body systems in terms of realistic nuclear interactions that reproduce the phase shifts. Only recently it became possible to perform quasi-exact ab initio calculations of the nuclei up to $A = 8 - 12$ with realistic interactions [1,2]. For bigger nuclei these calculations become exceedingly difficult.

Our aim is to perform approximate ab initio calculations of larger nuclei where the short-range central and tensor correlations induced by the realistic forces are explicitly introduced into the many-body state. For that we construct a unitary correlation operator C as a product of a radial correlator C_r and a tensor correlator C_Ω.

$$C = C_\Omega C_r \tag{1}$$

The correlated state $\big| \hat{\Psi} \big\rangle = C \big| \Psi \big\rangle = C_\Omega C_r \big| \Psi \big\rangle$ is obtained by a unitary mapping of the uncorrelated $\big| \Psi \big\rangle$.

The unitary correlation operator defines also a correlated interaction $\hat{H} = C^\dagger H C$, that can be considered as an effective interaction. Other observables can be correlated as well and the physical implications of the short-range correlations, for example on the nucleon momentum distributions, can be evaluated.

The radial correlator C_r [3], which shifts a pair of particles in the radial direction away from each other, uses as generator the radial momentum operator p_r together with a shift function $s(r)$ that depends on the distance of the two nucleons.

$$C_r = \exp\Big\{ -i \sum_{i<j} \frac{1}{2} \Big(s(r_{ij})\; p_{r\,ij} + h.a. \Big) \Big\} \tag{2}$$

The shift will be strong at short distances and will vanish at large distances.

The tensor force in the $S = 1$ channels depends on the spins and the spatial orientation $\vec{\hat{r}} = (\vec{r}_1 - \vec{r}_2)/(|\vec{r}_1 - \vec{r}_2|)$ of the nucleons via the tensor operator

$$S_{12}(\vec{\hat{r}}, \vec{\hat{r}}) = 3(\vec{\sigma}_1 \cdot \vec{\hat{r}})(\vec{\sigma}_2 \cdot \vec{\hat{r}}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) = 2\,\big(3(\vec{S} \cdot \vec{\hat{r}})^2 - \vec{S}^2\big)\ . \tag{3}$$

An alignment of $\vec{\hat{r}}$ with the direction of total spin $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ is favored energetically. The tensor correlator C_Ω, defined as

$$C_\Omega = \exp\Big\{ -i \sum_{i \neq j} \frac{3}{4} \Big(\vartheta(r_{ij})\, (\vec{\sigma}_i \vec{r}_{ij})(\vec{\sigma}_j \vec{p}_{\Omega\, ij}) + h.a. \Big) \Big\}\ , \tag{4}$$

achieves this alignment by shifts perpendicular to the relative orientation $\vec{r}_{ij}$. To achieve this the generator uses a tensor operator constructed with the orbital part of the momentum operator $\vec{p}_\Omega = \vec{p} - \vec{p}_r$. The r-dependent strength of the tensor correlations is controlled by $\vartheta(r)$.

The figure illustrates the action of C_r and C_Ω in the $S = 1, M_S = 1; T = 0$ channel of the ^{4}He two-body density $\rho^{(2)}_{S,T}(\vec{r})$. The arrow indicates the spin direction $\vec{S}$ and the correlation operator is determined for the Argonne V18 interaction. The left plot shows the density (bright areas = high densities) calculated in the uncorrelated state $\big| \Psi \big\rangle$ of four nucleons in the s-shell, here the nucleons are found with the highest probability at zero distance. The radially correlated state $C_r \big| \Psi \big\rangle$ yields the two-body density shown in the middle where a hole has been created to accommodate the repulsive core of the two-body potential.

The subsequent application of the tensor correlator $C_\Omega C_r \big| \Psi \big\rangle$ results in the correlations shown on the right. C_Ω, which acts only on the $S = 1$ part of a pair, moves probability perpendicular to $\vec{r}$ from the "equator" to the "poles". The spherical distribution transforms into an axially symmetric one with enhanced probability in regions where $\vec{r}$ and $\vec{S}$ are parallel, which implies more binding from the tensor interaction.

The correlated interaction $\hat{H}$ is used successfully with simple shell model and Fermionic Molecular Dynamics Slater determinants [4]. This allows us to perform calculations for all nuclei up to about $A = 50$. Although a single Slater determinant as many-body trial state is the most simple ansatz for the uncorrelated $\big| \Psi \big\rangle$ we obtain results very close to those of the quasi-exact methods.

[1] S.C. Pieper et al., PR **C64** (2001) 014001

[2] P. Navrátil et al., PR **C62** (2000) 054311

[3] H. Feldmeier et al., NP **A632** (1998) 61

[4] T. Neff, *Short-Ranged Central and Tensor Correlations in Nuclear Many-Body Systems*, PhD thesis, 2001