Low Momentum Nucleon-Nucleon Interaction and Fermi Liquid Theory^G

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Bogner, Kuo et al. [1] have constructed a low momentum nucleon-nucleon potential $V_{\rm low\,k}$ using folded-diagram techniques. Starting from a realistic nucleon-nucleon interaction, the hard momenta larger than a cutoff Λ are integrated out and renormalize an effective interaction $V_{\rm low\,k}$, such that the physical amplitudes at relative momenta smaller than Λ are preserved. For $\Lambda \lesssim 2\,{\rm fm}^{-1}$, they find that $V_{\rm low\,k}$ is unique, i.e. independent of the different short distance parts of the traditional potentials, and depends only weakly on the cutoff.

In normal Fermi systems, the low momentum quasiparticle interaction, which is characterized by the Fermi liquid parameters, is determined by a renormalization group (RG) fixed point. In this paper [2], we derive two novel relations between the Fermi liquid parameters of nuclear matter and the unique low momentum nucleon-nucleon interaction at $\Lambda = k_{\rm F}$. These relations connect the fixed point of the quasiparticle interaction to $V_{\rm low\,k}$ in the region where it depends only weakly on the cutoff.

The quasiparticle interaction, $\mathcal{F}$, parameterized in terms of Landau parameters F_l, etc., is given by

$$\mathcal{F}(\theta) = \sum_l (F_l + F_l'\,\boldsymbol{\tau}\cdot\boldsymbol{\tau}' + G_l\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}' + G_l'\,\boldsymbol{\tau}\cdot\boldsymbol{\tau}'\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')P_l(\cos\theta)\,, \quad (1)$$

where for brevity we have omitted the tensor terms and θ denotes the angle between the quasiparticles on the Fermi surface. The quasiparticle interaction is dimensionless in units of the density of states at the Fermi surface. The forward scattering amplitude, $\mathcal{A}$, is then obtained by solving the quasiparticle scattering equation. This sums all quasiparticle-quasihole reducible diagrams in the zero sound channel and one finds

$$\mathcal{A}(\theta) = \sum_l (B_l + B_l'\,\boldsymbol{\tau}\cdot\boldsymbol{\tau}' + C_l\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}' + C_l'\,\boldsymbol{\tau}\cdot\boldsymbol{\tau}'\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')P_l(\cos\theta)\,, \quad (2)$$

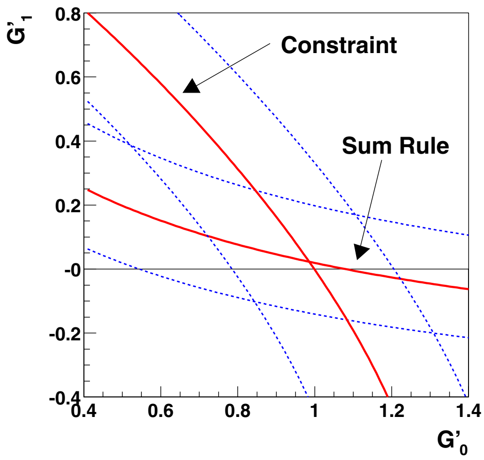
where $B_l = F_l/(1+F_l/(2l+1))$, etc. The Pauli principle requires that the forward scattering amplitude vanishes in singlet-odd and triplet-odd states for zero relative momentum, i.e., $\theta = 0$. This leads to the Pauli principle sum rules [3, 4]

$$\sum_l (B_l + B_l' + C_l + C_l') = 0 \quad (3)$$

$$\sum_l (B_l - 3\,B_l' - 3\,C_l + 9\,C_l') = 0. \quad (4)$$

The sum rules provide constraints on empirically determined Fermi liquid parameters. In an RG approach to Fermi liquids, they define an invariant under the decimation to the Fermi surface, which we show explicitly by deriving the RG equations in the particle-hole channels from the induced interaction.

Using the induced interaction of Babu and Brown [5], we derive two additional RG invariant constraints, which relate the Fermi liquid parameters of nuclear matter to the low momentum interaction in vacuum. We find in the singlet-even and triplet-even states [2]



$$\sum_l \{2F_l - B_l + 2F_l' - B_l' - 3\,(2G_l - C_l) - 3\,(2G_l' - C_l')\} = \mathcal{F}_{\rm driving}\,(S=0, T=1) \quad (5)$$

$$\sum_l \{2F_l - B_l - 3\,(2F_l' - B_l') + 2G_l - C_l - 3\,(2G_l' - C_l')\} = \mathcal{F}_{\rm driving}\,(S=1, T=0), \quad (6)$$

where the driving term $\mathcal{F}_{\rm driving}$ is given by the s-wave matrix elements of $V_{\rm low\,k}$ at $\theta = 0$.

Using these new constraints together with the Pauli principle sum rules, we can compute the large spin-dependent parameters, given the empirical values for the spin-independent parameters, F_0, F_1 and F_0'. In the figure we show the results for the spin-isospin parameters G_0' and G_1'. The dashed bands reflect the estimated errors in the constraints, due to the omitted parameters of higher l and errors in the input parameters. The resulting parameters are in fair agreement with empirical values as well as with microscopic calculations. Finally, the inclusion of tensor parameters into the analysis shows that these must be treated self-consistently in the induced interaction.

References

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