Mean-Field Instability of Trapped Ultracold Fermi Gases

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The success in cooling a cloud of fermionic ^{40}K atoms, kept in a magnetic trap, to temperatures significantly below the Fermi energy [1,2] opened a unique possibility to study the properties of dilute degenerate Fermi gases experimentally. One long-term goal is the observation of a BCS phase transition in these systems. The mechanism to create Copper pairs relies on an attractive interaction between the particles. At the same time these attractive interactions can cause a mean-field collapse of the metastable dilute Fermi gas towards high densities. Obviously a necessary prerequisite to reach the BCS transition is the stability of the normal Fermi gas.

We investigate the mean-field instability of the trapped Fermi gas in the framework of density functional theory. The energy density of the interacting multi-component Fermi gas is constructed in a mean-field picture using the Thomas-Fermi approximation. The interaction between the atoms is described by the s- and p-wave terms of the Effective Contact Interaction (ECI) [3,4], which depends on the s- and p-wave scattering length, a_0 and a_1, resp., as well as on the s-wave effective volume b_0. The ECI is constructed such that the exact two-body energy spectrum is reproduced by the expectation values of the ECI in mean-field type states.

The energy density of a single-component Fermi gas trapped in an external potential $U(\vec{x})$ reads

$$\mathcal{E}_1[\kappa(\vec{x})] = \frac{U(\vec{x})}{6\pi^2}\,\kappa^3(\vec{x}) + \frac{1}{20\pi^2 m}\,\kappa^5(\vec{x}) + \frac{a_1^3}{30\pi^3 m}\,\kappa^8(\vec{x}),$$

where $\kappa(\vec{x}) = \sqrt[3]{6\pi^2\rho(\vec{x})}$ denotes the local Fermi momentum and a_1 the p-wave scattering length. The first term originates from the external potential, the second one from the kinetic energy, and the third term from the p-wave interaction. The s-wave part of the interaction does not contribute in a system of identical fermions due to the Pauli principle.

For attractive p-wave interactions, i.e., negative p-wave scattering length $a_1 < 0$, the highest power of $\kappa(\vec{x})$ in the energy density has a negative coefficient. This implies that there is a maximum density up to which the system is stable. Beyond this density the energy decreases for growing density and the system collapses towards a high density configuration. For fixed particle number N we obtain the following local condition for the stability of the single-component Fermi gas [3,4]

$$-a_1\kappa(\vec{x}) \leq \sqrt[3]{3\pi}/2.$$

If this stability condition is violated anywhere in the trap then the system will collapse.

The energy density of a two-component Fermi gas, where we assume equal local Fermi momenta $\kappa(\vec{x}) = \kappa_1(\vec{x}) = \kappa_2(\vec{x})$ for both components, reads

$$\mathcal{E}_2[\kappa(\vec{x})] = \frac{U(\vec{x})}{3\pi^2}\,\kappa^3(\vec{x}) + \frac{1}{10\pi^2 m}\,\kappa^5(\vec{x}) + \frac{a_0}{9\pi^3 m}\,\kappa^6(\vec{x}) + \frac{\tilde{a}_1^3}{10\pi^3 m}\,\kappa^8(\vec{x})$$

with $\tilde{a}_1^3 = a_1^3 + b_0/3$. In this case both, s- and p-wave interactions contribute and influence the stability of the system. We obtain the following stability condition for the two-component Fermi gas [3,4]

$$-a_0\kappa(\vec{x}) - 2\,[\tilde{a}_1\kappa(\vec{x})]^3 \leq \pi/2.$$

For practical purposes this stability condition can be rephrased in terms of an upper limit in the number of particles $N = N_1 = N_2$ assuming a parabolic trapping potential with mean oscillator length $\ell = 1/\sqrt{m\omega}$ [3,4]. Figure 1 shows the logarithm of the maximum particle number of each component as a function of the s- and p-wave scattering length in units of the mean oscillator length.

Attractive s-wave as well as attractive p-wave interactions can cause a mean-field collapse. If both components are attractive then their combined action decreases the maximum allowed density or particle number for the stable configuration. If one component is attractive and the other repulsive then the repulsive one stabilizes the system, i.e., increases $N_{\max}$. New phenomena appear in the case of attractive s-wave and repulsive p-wave interactions. If the p-wave scattering length exceeds about $1/3$ of the s-wave scattering length (compare Fig. 1) the p-wave repulsion prevents a collapse completely, i.e., $N_{\max} \to \infty$. For p-wave repulsions slightly too weak to cause this absolute stabilization a novel high-density phase appears in the center of the trap if $N_{\max}$ is exceeded.

We conclude that p-wave interactions have an important influence on the stability and may thus be helpful on the way to Cooper pairing in trapped ultracold Fermi gases.

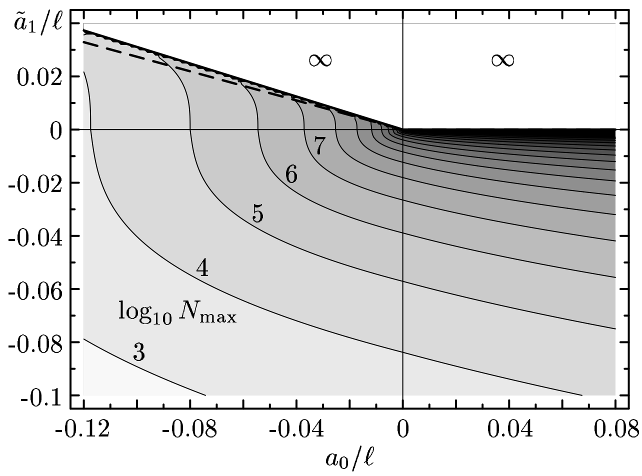

Fig. 1: Contour plot of the logarithm of the maximum particle number $N_{\max}$ in a two-component Fermi gas as function of the s- and p-wave scattering lengths, a_0/ℓ and $\tilde{a}_1/\ell$, where ℓ is the mean oscillator length of the trap.

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[3] R. Roth, H. Feldmeier; e-print: cond-mat/0102416.
[4] R. Roth, H. Feldmeier; J. Phys. B 33 (2000) 787.
see also: http://theory.gsi.de/˜trap/