Nonpertubative renormalization flow and infrared physics

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Renormalization group flow equations have proved to be a good tool to analyze the dynamics of strong interaction in the nonperturbative region. They have been successfully applied to effective mesonic models in a thermal environment and revealed detailed insight into the chiral phase transition [1]. The basic aim of the renormalization group treatment is to systematically integrate out quantum and/or thermal fluctuations with momenta above a certain cutoff scale k and include them into the couplings of an effective action.

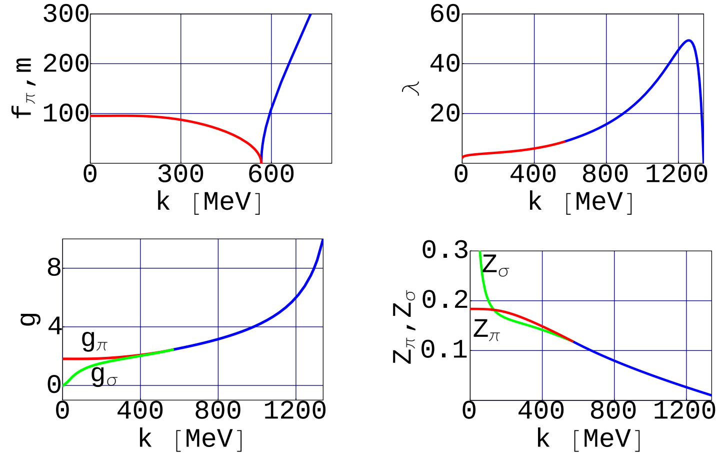
We are interested in the low energy theory of strong interaction resulting from the integration of high momentum modes. Assuming a dominant one gluon exchange or a instanton vacuum, gluonic degrees of freedom can be absorbed into an instantaneous four fermion interaction described by the NJL-model. Our analysis shows that the standard solution of the NJL-model and the flow of the linear σ-model, which we are using, coincide in the large N_C-limit, where the dynamics is dominated by fermion loops. In the full solution though, the additional dynamics of the mesonic degrees of freedom becomes important and yields an improved low energy behaviour compared to the NJL-model. The linear σ-model describes a massless two flavour quark field q interacting with chiral fields $\Phi = (\sigma, \vec{\pi})$ and is believed to give a valid description of chiral dynamics at scales $\leq$ 1GeV. It has the action

$$S_{UV} = \int d^4x \left[\bar{q} \left(i\partial\!\!\!/ - g \left(\sigma + i\vec{\tau}\vec{\pi}\gamma_5 \right) \right) q + \frac{1}{2} \left(\partial\Phi \right)^2 - U(\Phi^2) \right],$$

with a general O(4) symmetric potential $U(\Phi^2)$. From this, we compute the effective action in a one loop approximation using Schwinger proper time regularization [2]. The resulting expression is truncated to second order derivative in order to include the relevant terms for the flow. This inclusion of higher order terms, results in a splitting in the dynamics of the massive σ-mode and the massless Goldstone-bosons. By a renormalization group improvement the one loop expressions are turned into nonperturbative flow equations. The resulting flow for the most important parameters is plotted below. An interesting feature is, that like the four boson coupling λ also the Yukawa coupling g_σ vanishes in the infrared. Therefore, aside from providing a broken vacuum, the σ decouples from the dynamics and leaves the pions as the only dynamic particles. We obtain the resulting infrared effective action

$$\Gamma_{IR} = \int d^4x \Big[\bar{q} \left(i\partial\!\!\!/ - m_q - i g_\pi \vec{\tau}\vec{\pi}\gamma_5 + \frac{g_\pi}{f_\pi}\pi^2 \right) q - \frac{1}{2} \left(\partial\vec{\pi} \right)^2 + \text{derivative couplings} \Big].$$

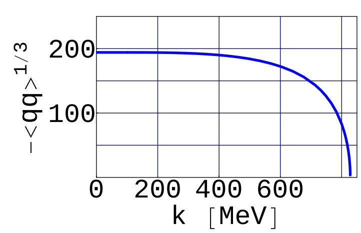
Our approximation obeys the chiral Ward identities and ensures that the $\pi\pi$-scattering amplitude vanishes at tree level due to a $\bar{q}q\pi^2$-contact term that cancels the 1π-contributions in the IR.



Once the flow equations for the linear σ-model are written down, the chiral order parameter $\langle \bar{q}q \rangle$ can be calculated from the partition function. Using the same cutoff function as for the flow equations one obtains in the local potential approximation

$$\partial_k \langle \bar{q}q \rangle = - \frac{N_c g k^5 \phi_k}{2\pi^2 \left(k^2 + g^2 \phi_k^2 \right)^2} .$$

Starting in the symmetric regime where the order parameter is zero, we find a value of $\langle \bar{q}q \rangle = -(194\text{Mev})^3$, as shown below.



It is now tempting to derive a flow equation for the spectral density $\rho(\lambda)$ of the Dirac operator. This may be done by establishing a relation between $\langle \bar{q}q \rangle (\lambda)$ and $\rho(\lambda)$ similar to the Banks-Casher relation. It would be interesting to compare the results with recent data available from lattice QCD.

Another promising idea is to connect our cutoff parameter to a physical momentum scale and continue our Euklidian flows to the full complex plane. By this, it would be possible to compute spectral functions of mesonic resonances and make a direct connection to low energy hadron phenomenology.

References

[1] J. Berges, N. Tetradis and C. Wetterich, to appear in Phys. Rep., hep-ph/0005122

[2] B.-J. Schaefer and H.-J. Pirner, Nucl. Phys. **A660** (1999) 439, nucl-th/9903003