Thermal Boson Expansion

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Based on recent progress in the application of thermo field dynamics (TFD) [1] to thermal many-body systems, several authors [2, 3, 4] have considered a consistent thermal boson expansion (TBE). In this regard two points of view have been taken [2]. The first (path I) consists of a bosonization of the original degrees of freedom of the system, substituting for these ideal boson images. The thermalization is then achieved by doubling those newly introduced bosons as according to TDF prescription. The second possibility (path II) proceeds on the other hand via a thermalization of the system by doubling the original degrees of freedom and a subsequent bosonization of the entire new system. The two paths do not lead to the same results. Moreover, when applied to the Lipkin model, a closer look shows that the choice of path I implies that the thermal density of states are of bosonic type although the original system is purely fermionic.

To circumvent this problem one may choose path II where the thermal density of states is of fermionic type, since the thermalization is performed on the original fermions. However, inconsistencies related to the quasi-particle energies defining the thermal density of states, emerge. The latter are usually taken as solutions of a Hartree-Fock-Bogoliubov (HFB) approximation which, in most cases, leads to a dynamical mass generation. For massless modes, such as Goldstone modes, this gives the wrong solution. On the other hand, a mean-field description that is compliant with the symmetry requirements, is the Hartree-Bogoliubov (HB) approximation which can be obtained after a bosonization à la Holstein-Primakoff (HP). Therefore, when considering the symmetry constraints, it is rather path I that is favored. Amendments are, however, needed in order to reconcile it with the requirements of the statistics as explained earlier.

It was shown in [5] that a Boson expansion approach that treats on equal footing pair- as well as single-particle mapping offers a simple solution to the problems outlined above. For that matter the extended form of the bosonic HP mapping has been proposed in [6] which accommodates single-boson and boson-pair mappings:

$$\begin{aligned} (\vec{a}\vec{a})_I &= \mathcal{G}_N(n,m)A\ , \qquad (\vec{a}^+\vec{a})_I = 2n+m\ , \\ (a_i)_I &= \mathcal{G}_N(n,m)\Gamma_N(m)\alpha_i + 2\alpha_i^+ A\Gamma_N(m)\ , \\ (\vec{a}^+\vec{a}^+)_I &= (\vec{a}\vec{a})_I^+\ , \qquad (a_i^+)_I = (a_i)_I^+\ , \end{aligned} \tag{1}$$

where N is an integer, $n = A^+A$, $m = \sum_i \alpha_i^+\alpha_i$, and Γ_N is given by

$$\mathcal{G}_N(n,m) = \sqrt{2N+4(n+m)}\ , \tag{2}$$

$$\Gamma_N(m) = \left[\frac{m+N-2}{2(2m+N)(2m+N-2)}\right]^{\frac{1}{2}}\ . \tag{3}$$

Thus, instead of the original bosons a_i, one has an ideal boson α_i which, as was shown in ref. [6], can accommodate the symmetry requirements. This is at the expense of introducing a power series in an auxiliary boson A. The thermalization of the system is then undertaken in a consistent way by using the TFD formalism. The time-translation operator, $\mathcal{H} = H - \tilde{H}$, of the system is obtained as usual by considering the tilde conjugate of all operators such as A and α_i among others. However, the independent thermal quasiparticle representation is obtained by rotating only the ideal bosons α_i, α_i^+ and their tilde conjugate (t.c.), via a unitary thermal Bogoliubov transformation

$$\alpha_i^+ = u(T)\gamma_i^+ + v(T)\tilde{\gamma}_i\ , \tag{4}$$

into the thermal quasiboson operators γ_i, γ_i^+, and their t.c. We insist here on the fact that the bosons $A, \tilde{A}$ need not be transformed since they are only auxiliary modes. This point of view is different from those adopted in all earlier works [2, 3, 4].

For fermionic systems, the situation is rather similar to the bosonic case. The extended form for the fermionic HP mapping proposed by Marshalek [7] is given by

$$\begin{aligned} (J_z)_I &= \frac{1}{2}n_f + B^+B\ , \\ (J_+)_I &= B^+\sqrt{N-(B^+B+n_f)}\ ; \quad (J_-)_I = (J_+)_I^+ \\ (c_{2p})_I &= N^{-1}\left(\sqrt{N-(B^+B)}\,a_{2p} + B\,a_{1p}\right) \\ (c_{1p})_I &= N^{-1}\left(\sqrt{N-(B^+B)}\,a_{1p} - B\,a_{2p}\right) \\ n_f &= \sum_{p=1}^{N}(a_{2p}^+a_{2p} - a_{1p}^+a_{1p})\ . \end{aligned} \tag{5}$$

where B^+ and a_{ip}^+ are ideal boson and fermion operators, respectively. It allows a consistent mapping of pairs and single-fermion states. The thermalization is again obtained following the amended path I. Thus one introduces as previously the thermal Bogoliubov transformation which rotates only the ideal fermions and their tilde transform, such that

$$a_{ip}^+ = x_i\beta_{ip}^+ + y_i\tilde{\beta}_{ip}\ , \tag{6}$$

while the auxiliary bosons B are left unaltered. Obviously our procedure cures the problems of path I that were encountered in ref. [2] regarding the fermion statistics [5].

References

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