Low velocity stopping in a strongly magnetized plasma : a guiding center approach

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One witnesses presently an intense theoretico-numerical activity in the topic of ion stopping in arbitrarily magnetized hot and dense plasmas [1].

We suggest here a novel avenue to the extreme magnetization limit with electron Larmor radius $\ll$ electron Debye length. We intend to combine the well-know Guiding Center (GC) approximation [2, 3] with Einstein relations connecting ion mobility μ to its diffusion coefficient $D_\perp \perp$ magnetic field **B**. Very recently, Dufty *et al.* [4] have proposed

$$Y(V_B \to 0) \equiv \lim_{V_B \to 0} \frac{S(V_B)}{V_B} = \mu^{-1} = k_B T . D_\perp^{-1}, \qquad (1)$$

for the ratio of ion stopping power $S(V_B)$ at vanishingly small ion projectile velocity V_B , to V_B. Eq.(1) highlights ion projectile energy losses $\perp$ **B**, which are the dominant ones, especially in GCA.

The Green-Kubo formalism for the linear response theory provides the particle diffusion coefficient $D_\perp$ across an external magnetic field **B** as

$$D_\perp = \frac{c^2}{B^2} \int_0^\infty d\tau \langle \overrightarrow{E}(\tau) \cdot \overrightarrow{E}(0) \rangle \qquad (2)$$

in terms of an equilibrium canonical average of the two-point autocorrelation function for fluctuating electric fields.

The GC approach is tantamount to describe the spatial diffusion of the test ion through small increment of $\mathbf{x}(\tau)$ and the velocity-space diffusion along **B** via the equations of motion

$$\begin{aligned} \frac{d\overrightarrow{x_\perp}}{dt} &= c \frac{\overrightarrow{E_\perp}(t) \times \overrightarrow{B}}{B^2} \\ \frac{d\overrightarrow{v_\parallel}}{dt} &= \frac{e_i}{m_i} \overrightarrow{E_\parallel}(t) \end{aligned} \qquad (3)$$

The first of two describes the $\mathbf{E} \times \mathbf{B}$ GC drift wich is a good approximation in the limit of a zero Larmor radius, i.e., $(\Omega_i/\omega_{pi})^2 \gg 1$. When turning our attention to the low-frequency and long-wavelength parts of the electric field spectra, it is expected that a free streaming of the particles along **B** acts to destroy the electric field correlation between two-time points.
Moreover, large k (short r) cutoffs are not required in present formulation, because the various coulomb interactions (e-i, e-e) take care of uncertainty principle (Heisenberg) and electron-electron Pauli repulsion through [5]

$$\begin{aligned} u_{ee}(r) &= \tfrac{e^2}{r}(1 - e^{-r/\overline{\lambda}}) \\ &\quad + k_B T \cdot ln2 \exp\left[-\tfrac{1}{\pi ln2}(r/\overline{\lambda})^2\right] \\ u_{ee}(r) &= \tfrac{Ze^2}{r}(1 - e^{-\sqrt{2}r/\overline{\lambda}}) \\ u_{ii}(r) &= \tfrac{Z^2 e^2}{r} \end{aligned} \qquad (4)$$

with $\overline{\lambda}$, electron thermal wavelength (De Broglie).
$D_\perp$ is then expressed by

$$\frac{D_\perp}{\omega_{pi}\lambda_D^2} = \frac{1}{(2\pi)^2} \frac{\omega_{pi}^2}{\Omega_i^2} \Lambda \dot{z}(\infty) \qquad (5)$$

in terms of $\Lambda = 1/n\lambda_D^3$, λ_D Debye length, and

$$\dot{z}(\infty) = \frac{1}{2}[\alpha + (\alpha^2 + 4\beta)^{1/2}] \qquad (6)$$

α and β denote combinations of particle structure factors at equilibrium. The B-dependance is restricted to β.

β is proportional to $(\Omega_i/\omega_{pi})^2$, that is, to B^2. This implies that, whenever $4\beta \gg \alpha^2$, $\dot{z}(\infty) \sim \sqrt{\beta}$ and that, in the opposite limit, $\dot{z}(\infty) \sim \alpha$. Correspondingly $D_\perp$ is proportional to 1/B (Bohm-type) in the former case and turns out to be in the classical type in the latter. Between the two, i.e., when $\alpha^2 \sim 4\beta$, a hybrid diffusion takes place. In order to examine quantitatively these situations, extensive numerical analysis have been carried out. For fixed values of L (characteristic plasma size) and n (total number density), we have varied $P = (2\pi\lambda_D/L)^2$ which amount to varying T (temperature).

Provisional conclusions include the novel limits

$$\begin{aligned} Y(V_B \to 0) &\sim B, \qquad \textit{Bohmlike} \\ Y(V_B \to 0) &\sim B^2, \qquad \textit{Classical} \end{aligned} \qquad (7)$$

with a novel (n,T) dependance which is under active scrutiny.

References

[1] C. Cereceda *et al.*, *Phys. Plasmas* **7**, 2884 (2000) and **12**, 022102 (2005).
[2] G. Vahala, *Phys. Fluids* **16**, 1876 (1973).
[3] Y. Furutani, Y. Oda, C. Deutsch and M.M. Gombert, *Phys. Rev.*, **26A**, 2913 (1982).
[4] J.W. Dufty, B. Talin and A. Calisti, *Adv. Qtum Chemistry*, **46**, 293 (2004).
[5] C. Deutsch, *Phys. Lett.*, **A60**, 317 (1977) and C. Deutsch, M.M. Gombert and H. Minoo, *Phys.Lett.*, **A66**, 381, (1978).