The LAPLAS experiment: Rayleigh – Taylor instability in elastic solids

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LAPLAS (Laboratory of Planetary Sciences) experiment [1] is one of the most interesting experimental schemes proposed for the study of equation of state and transport properties of high energy density matter in the framework of the FAIR (Facility for Antiproton and Ion Research) project [2]. This experiment involves a cylindrical implosion of a thick shell of Au or Pb driven by a heavy ion beam with annular focal spot. The region of the shell heated by the ion beam (the absorber) expands accelerating the pusher layer that surrounds the sample in the internal axial region. During this process in which the hot and less dense absorber accelerates the denser pusher, the conditions for the growing of the Rayleigh –Taylor instability (RTI) arise. Therefore, this is an issue of possible concern that is presently under investigation. The classical RTI is a very well known phenomenon that has been extensively studied during the last 50 years but the LAPLAS experimental conditions are quite far from such a classical situation. In fact, during the implosion the pusher remains in solid state retaining its elastic and plastic properties whilst the absorber is melted remaining in liquid state. So, the elastic – plastic properties of the pusher as well as the liquid internal frictional forces (viscosity) will affect the evolution of RTI.

The RTI physics in solids is significantly determined by the elastoplastic properties of the material and exhibit a series of complex phenomena that are note well understood yet. For this, it results of great help to analyze situations involving materials with pure elastic or plastic properties. In this regards, the problem of the linear instability in perfectly elastic solids has been studied in the past by using different methods. On one side, simple models are very suitable for describing more physics as, for instance, the effect of the transient phase between the initial conditions and the asymptotic regime, the effect of the Atwood number A_T [$A_T= (\rho_2-\rho_1)/ (\rho_2+\rho_1)$, where ρ_2 is the density of the solid and ρ_1 is the density of the lighter solid or fluid below it] [3]. However, such models have given so far only a qualitative description of the RTI whislt quantitative results are needed for an adequate design of the experiments. Exact results for the perfectly elastic case were obtained in Ref [4] but this work is limited to $A_T =1$ and it is so involved mathematically to make difficult to generalize it for including more physics. Recently, a new exact solution has been presented in Ref.[5] that can deal with arbitrary values of A_T but it is based in a normal mode analysis so that it is limited to the asymptotic regime. Although limitations of the exact solutions of Ref.[4] and [5] they are very valuable for testing the accuracy of more general but approximate models.

Here we present an analytic model that can describe the transient initial phase as well as the asymptotic regime[6]. Besides, the model is very simple and accurate so that more physics can be included conveniently as, for instance, the effect of arbitrary A_T. This model may be the first step to a more complete elastoplastic theory yet inexistent.

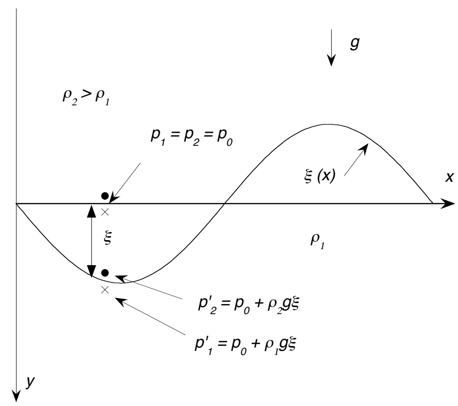

Fig. 1 Diagram of the perturbed interface

Let us show first how the classical results can be obtained from this model [6]. For this we consider two inviscid fluids in a gravitational field g as in Fig.1. If the interface is perfectly planar and it is in equilibrium, the fluid elements on each side of the interface immediately above and below it, respectively, have the same pressure $p_1=p_2=p_0$. If we introduce a small perturbation $\xi(x)$ in the surface, so that these fluid elements originally at $y=0$ are translated to a lower position $y = \xi(x)$, the pressure of the fluid elements will increase as they are in a deeper place within the fluid. However, pressure in the denser fluid increases more than pressure in the lighter one:

$$p_1' = p_0 + \rho_1 g\xi, \; p_2' = p_0 + \rho_2 g\xi \quad (1)$$

Therefore, a pressure difference $\Delta p = (\rho_2-\rho_1)g\xi$ is created which tends to deform the interface fatherly. According to the Newton second law, the equation of motion of the interface can be written as follows:

$$m\ddot{\xi} = \Delta p A \quad (2)$$

where A is the area of the interface and m is the mass of both fluids involved in the motion. Since in the RTI we have to deal with surface modes, we can assume that in the linear regime only the fluid within a distance equal to $1/k$ participates in the motion ($k= 2\pi/\lambda$ is the perturbation wave number and λ is the wave length). Therefore, we get:

$$m = m_1 + m_2 = \rho_1 \frac{A}{k} + \rho_2 \frac{A}{k} \quad , \quad (3)$$

where m_1 and m_2 are, respectively, the masses of the light and heavy fluids that participate in the motion. Thus the equation of motion reads:

$$(\rho_2 + \rho_1)\frac{\ddot{\xi}}{k} = (\rho_2 - \rho_1)g\xi \quad . \quad (4)$$

As it is well known this equation can easily be solved to get the classically exponential growth rate $\gamma = (A_T kg)^{1/2}$.

This very simple model clearly shows how the interface motion is driven by the buoyancy force $(\rho_2-\rho_1)g\xi A$. If other forces F_i are present, for example due to viscosity, surface tension, elasticity, etc, they must be added to the equation of motion [6]:

$$(m_1 + m_2)\ddot{\xi} = (\rho_2 - \rho_1)g\xi A + \sum_i F_i \quad . \quad (5)$$

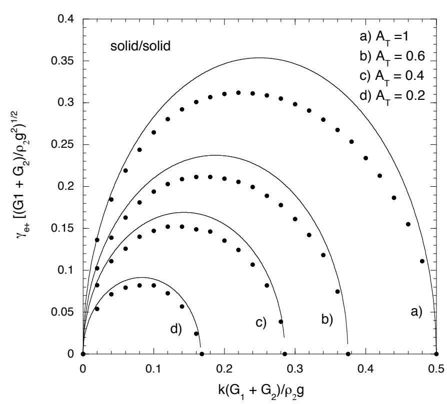

Fig. 2 Asymptotic growth rate as a function of the perturbation wave number for a solid/solid interface and for several values of the Atwood number. Dots are the results of Ref.[5].

So, provided that we can calculate the corresponding forces F_i acting on the interface, this equation will describe the evolution of the interface. For the particular cases of viscous fluids and elastic solids, the forces that act on the interface can be calculated approximately from the constitutive relationships of the material by assuming the velocity field of an inviscid fluid [6]. Then, for a viscous fluid we get:

$$F_v = -2\mu k\dot{\xi}A \quad , \quad (6)$$

where μ is the fluid viscosity. In the same way, for an elastic (Hookean) solid we obtain:

$$F_e = -2Gk(\xi - \xi_0)A \quad , \quad (7)$$

where G is the shear modulus of the material and ξ_0 is the perturbation initial amplitude. Using these results we can analytically solve the equation of motion for the cases of solid/solid and solid/fluid interfaces. In Figs. 2, 3 and 4 we show the asymptotic growth rate as a function of the perturbation wave number for different cases. Dots corresponds to the exact results of Ref.[5]. The model also describes the transient phase between the initial conditions and the asymptotic regime for arbitrary Atwood numbers. For the particular case of A_T *=1* treated in Ref.[4] the result of the present model are also in excellent agreement.

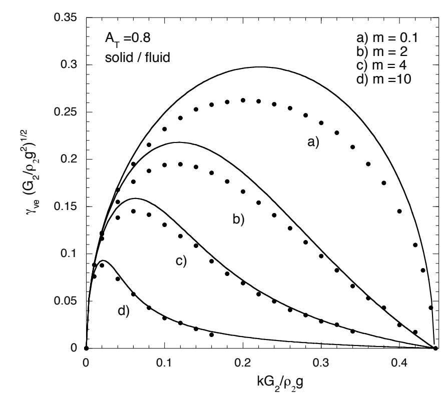

Fig. 3. Asymptotic growth rate as a function of the perturbation wave number for a solid/fluid interface, for an Atwood number 0.8 and for several values of the dimensionless viscosity parameter m=$[\mu_1 g(\rho_2/G_2^3)^{1/2}]^{1/2}$. Dots are the results of Ref.[5].

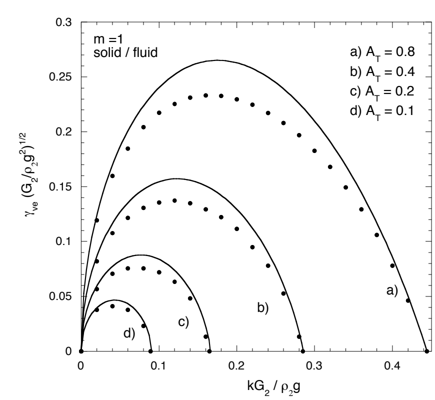

Fig. 4. Asymptotic growth rate as a function of the perturbation wave number for a solid/fluid interface, for several Atwood numbers and for a dimensionless viscosity parameter m=$[\mu_1 g(\rho_2/G_2^3)^{1/2}]^{1/2}$=1. Dots are the results of Ref.[5].

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