

Instabilities in elastic accelerated solids

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When a solid plate is accelerated by an external pressure p_0 an instability similar to the Rayleigh-Taylor one appears if the plate is not perfectly flat. Due to the elastoplastic properties of the material the growth of the instability is lower than in a pure fluid case. In a very simple interpretation, the pressure difference across the irregular surface provokes the instability that can be stabilized by the elastic force of the body. Then, if the elastic regime is exceeded, the material begins to flow plastically and becomes unstable but the development of the instability is smoother than in the ideal situation. So it is important to determine which values of the elastic properties can produce this stabilizing mechanism.

One of the most accepted approaches to treat the instability of accelerated elastic solids is to make a correspondence between the viscosity μ and the elastic shear modulus G , through $\mu = G/n$ assuming a factor e^{nt} for all quantities [1]. Other approach assumes an isothermal, isotropic and hyperelastic material [2]. According to the later one, the growth rate n depends on the value of p_0/G , which is not reflected by the first approach. However, in the limit of small wavelengths compared to the thickness of the plate $kh \rightarrow \infty$, both approaches predict the same critical wavelength given by:

$$\frac{p_0}{G} = \frac{4\pi h}{\lambda} \quad (1)$$

On the contrary, in the first approach mentioned [1] the temporal evolution of the perturbation $\xi(t)$ is exponential:

$$\xi(t) = \xi_0 e^{nt} \quad (2)$$

while in the second approach [2] it is given by the next equation:

$$\xi(t) = \xi_0 \left[1 + \sum_n \frac{B(B+C)}{n(\partial D/\partial n)} (e^{nt} - 1) \right] \quad (3)$$

where the sum is extended over the infinite growth rates predicted by this model. The values of B , C and D , depend on k , g , G , ρ , h and n and can be found in Reference [2].

If we represent $\xi(t)$ for a stable situation (Fig. 1) we can see that both solutions are oscillatory, but the mean value and the amplitude given by equation (3) are much higher than the initial perturbation ξ_0 .

We have performed some numerical simulations with the explicit version of the ABAQUS finite element code. The numerical model we have used is described in detail in Reference [3]. The material counterpart of the model is composed by a volumetric part, which is defined by a Mie-

Grüneisen equation of state and a perfectly elastoplastic model for the deviatoric part, defined by the shear modulus G and the yield stress Y . We have selected a shear modulus that leads to a stable pattern according to equation (1).

To accelerate the plate we apply an external pressure at the perturbed surface that takes some time t_d to reach the maximum value p_0 . We have found that, for a pure elastic calculation ($Y = \infty$) the mean value is independent of the rise time t_d and that it is very close to the mean value predicted by the approach of Reference [2]. However, the amplitude of the oscillation depends on t_d as it is shown in Fig. 2. So if we take into account the plastic part of the model, we can find a very different behaviour with the same material parameters in function of the rise time. This fact is clearly shown in Fig. 2. With the short t_d the material is unstable while with the long t_d is stable. The explanation is that the higher oscillations produce plastification of the material, which leads to an unstable behaviour.

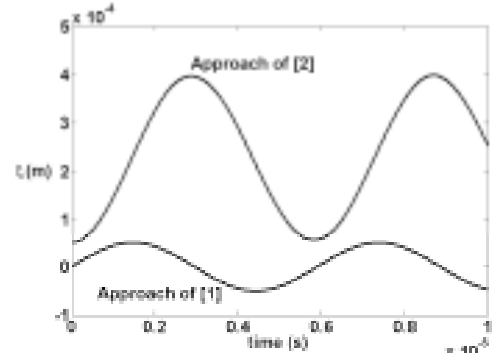


Fig. 1 Analytical solution for $\xi(t)$

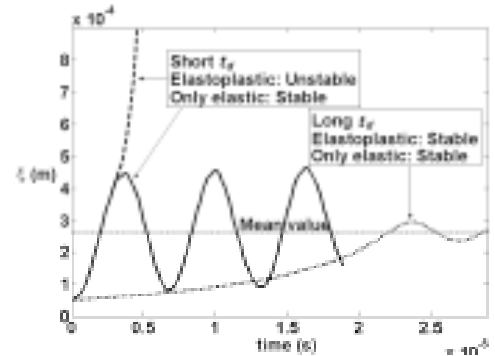


Fig. 2 Numerical solution for $\xi(t)$

References

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