The Z-pinch snowplow model revisited

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Z-pinches are an excellent way to produce and confine high-temperature plasmas. These plasmas generate copious x-rays radiation that have numerous applications such as the lithography and microscopy, and also in inertial fusion for driving high-temperature radiation cavities (hohlraums). Z-pinch is formed by the implosion of a thin current sheath driven by the magnetic pressure. The current sheath is created on the inner surface of a cylindrical discharge tube of radius r_0 that separates the electrodes on which a large voltage is suddenly applied. The simplest way to describe the Z-pinch formation is by means of the snowplow model [1]. It has been widely used for the experimental design and, more recently, for the study of possible methods for stabilizing the hydrodynamic instabilities that affect the implosion of the current sheath and the final pinch uniformity. This model is able to predict quite accurately the implosion time but is inadequate for describing the motion of the preceding shock wave as well as the plasma behaviour after the pinch time. This is because it assumes that the magnetic piston and the shock wave launched into the internal gas are lumped together into a plasma shell of zero thickness. Later models aimed to overcome these shortcomings has been proposed along the time, but no-one has been able to predict the pinch time as accurately as the classical snowplow model [2,3]

We have developed a new model which combines the description of the magnetic piston given by the snowplow model with the slug model for the motion of the shock wave driven by the piston. In this way the model keeps the accuracy of the snowplow model in the prediction of the pinch time whilst it allows for the calculation of the plasma conditions at peak compression.

We have assumed the simplest situation in which the cylindrical current sheath has an initial radius r_0 (at $t=0$) and the implosion is driven by a time varying external current $I(t)$. The current sheath is imploded by the magnetic force $(\boldsymbol{J}\times\boldsymbol{B})/c$, where $\boldsymbol{J} = J_z\,\boldsymbol{e}_z$ is the current density and $\boldsymbol{B} = B_\theta\,\boldsymbol{e}_\theta$ is the magnetic field created by this current (c is the light speed, and $\boldsymbol{e}_z$ and $\boldsymbol{e}_\theta$ are the unit vectors in cylindrical coordinates). We describe the motion of the magnetic piston by means of a snowplow model and for this we assume that the total current $I(t)$ is concentrated in a thin shell close to the piston position $r_p(t)$.

We consider that a strong shock wave is launched into the internal gas due to the motion of the current sheath so that the kinetic pressure between the shock and the current sheath is $p_s = [2/(\gamma+1)]\rho_0\,\dot{r}_s^2$ (ρ_0 is the initial density in the discharge tube, $\gamma = 5/3$). Then, we obtain the equation of motion of piston by integrating the fluid momentum conservation equation across the thickness of the current sheath:

$$\frac{d}{dt}\left[m(t)\frac{dr_p}{dt}\right] = -\frac{2I(t)^2}{c^2 r_p} + 2\pi r_p p_s\,, \quad m(t) = \rho_0\pi(r_0^2 - r_0^2)$$

where $m(t)$ is the shell mass per unit of length and it is equal to the mass swept at the time t. It may be worth to notice that the term corresponding to the magnetic force here is a factor two larger than the force due to the magnetic pressure $B_\theta^2/8\pi$ usually considered in the classical snowplow model. Besides the previous equation includes the counter-pressure due to the plasma between the shell and the shock. The pressure p_s depends of the shock motion and it can be described by the following slug model equation [2]:

$$\ddot{r}_s = -\frac{\gamma\dot{r}_s}{r_p^2 - r_s^2}\left(r_p\dot{r}_p - \frac{2}{\gamma+1}r_s\dot{r}_s\right)$$

where r_s is the instantaneous position of the shock wave.

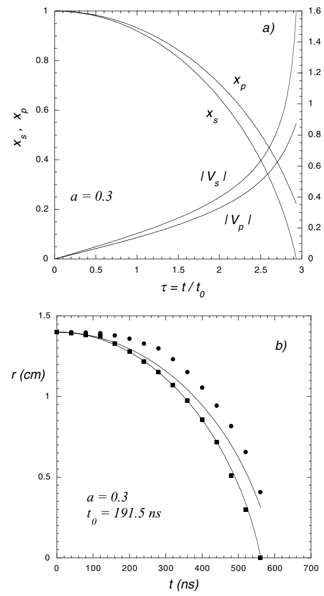

Fig. 1 a) Dimensionless shock and piston trajectories and velocities for a=0.3. b) Shock and piston trajectories given by the model (full lines) and by the simulations of Ref.[4] (dots).

In Fig.1 we show the results of the model and comparisons with numerical simulations for $I = I_0\,\sin(a\tau)$ and $a = 0.3$ ($\tau = t/t_0$, $a = 2\pi t_0/T$, $t_0 = (\rho_0\pi r_0^4 c^2/2I_0^2)^{1/2}$, and T is the period). From these results the model also allows for calculating the average density and temperature at peak compression in good agreement with the numerical simulations.

References.

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