The Evolution Operator for "Hydrogen and Strong Laser Field"

S. Gordienko and J. Meyer–ter–Vehn (Max-Planck-Institut für Quantenoptik, Garching)

Here we present an analytical three-dimensional solution of the time-dependent Schrödinger equation for a hydrogen-like atom in a strong laser field. The non–perturbative method developed here is based on the special property of the Coulomb potential, $\Delta(1/r) = 0$ for $r > 0$.

We are looking for the solution of the Schrödinger equation for a hydrogen-like atom in a strong electromagnetic field

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + U(\mathbf{r}) + e\mathbf{E}(t)\cdot\mathbf{r}\right)\psi, \tag{1}$$

with the initial condition

$$\psi(t = t_0, \mathbf{r}) = \int \psi_0(\mathbf{p}) \exp(i\mathbf{p}\cdot\mathbf{r}/\hbar) d^3\mathbf{p}, \tag{2}$$

and $U(r) = -Ze^2/r$. We will seek a solution of Eqs (1), (2) of the type of $\psi = \exp(iS/\hbar)$, where

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + U(\mathbf{r}) + e\mathbf{E}(t)\cdot\mathbf{r} - i\hbar\frac{\Delta S}{2m}, \tag{3}$$

and $S(t = t_0) = -i\hbar \ln \psi(t = t_0, \mathbf{r})$.

For finding the Green function of the problem, we first need solutions satisfying the initial condition

$$S(t = t_0, \mathbf{r}) = \mathbf{p}\cdot\mathbf{r} \tag{4}$$

instead of Eq. (2). Solutions of Eq.(1) for a general initial state $\psi_0(\mathbf{p})$ then require an additional Fourier transform.

It is convenient to represent the action S in the form $S = S_\mathbf{p} + \sigma_\mathbf{p}$. Here $S_\mathbf{p}$ represents the Volkov solution describing the electron in the laser field without the Coulomb interaction. One finds $S_\mathbf{p} = \mathbf{g}_\mathbf{p}(t)\cdot\mathbf{r} + G_\mathbf{p}(t)$ with

$$\mathbf{g}_\mathbf{p}(t) = \mathbf{p} + \frac{e\mathbf{A}(t)}{c},\; G_\mathbf{p}(t) = -\frac{1}{2m}\int_{t_0}^{t}\left(\mathbf{p} + \frac{e\mathbf{A}(\tau)}{c}\right)^2 d\tau.$$

The additional phase $\sigma_\mathbf{p}$ arising from Coulomb interaction then satisfies

$$-\frac{\partial\sigma_\mathbf{p}}{\partial t} = \frac{\mathbf{g}_\mathbf{p}(t)\cdot\nabla\sigma_\mathbf{p}}{m} + U(\mathbf{r}) - i\hbar\frac{\Delta\sigma_\mathbf{p}}{2m} \tag{5}$$

with $\sigma_\mathbf{p}(t = t_0) = 0$ under the condition $(\nabla\sigma_\mathbf{p})^2 \ll \mathbf{g}_\mathbf{p}^2$. It applies to the high-field case for sufficiently small time-steps such as a laser period. The approximate solution of Eq.(5) is

$$\sigma_\mathbf{p} = -\int_{t_0}^{t} U\left(\mathbf{r} - \frac{1}{m}\int_{\tau}^{t}\mathbf{g}_\mathbf{p}(\tau')\,d\tau'\right) d\tau,$$

as one can verify by direct insertion. Notice that the term $-i\hbar\Delta\sigma_\mathbf{p}/2m$ cancels for $U \propto 1/r$. It can be shown that $(\nabla\sigma_\mathbf{p})^2 \ll \mathbf{g}_\mathbf{p}^2$ is fulfilled if at least one of the conditions $Ze^2/r < \max(a^2mc^2, |\mathbf{p}|^2/2m)$ or $Ze^2/r^2 \ll eE$ is valid. In addition, one has to satisfy the initial conditions given by Eq. (4). At this point, difficulties can occur in the region where the electric field of the laser is the same or larger than the electric field of the ion. More careful analysis gives two additional restrictions that should be satisfied to have electrons with the laser field quiver velocity at the vicinity of the ion

$$a > \frac{4\pi mZe^2}{\lambda p^2},\; \tau_p > \frac{m}{p}\sqrt{\frac{Zr_e\lambda}{2\pi a}},$$

where τ_p is laser pulse duration and $r_e = e^2/mc^2$, $\lambda = 2\pi c/\omega$.

The Green function we have been looking for is then obtained in the form

$$G(t, \mathbf{r}|t_0, \mathbf{r}') = \int \exp\left(i\frac{S(t, \mathbf{r}, \mathbf{p}) - \mathbf{p}\cdot\mathbf{r}'}{\hbar}\right)\frac{d^3\mathbf{p}}{(2\pi\hbar)^3}.$$

It can be used to derive important properties of laser induced bremsstrahlung (see next contribution), high harmonics generation such as non-exponential spectral roll-off, and photo-electron spectra above threshold ionization. It should be understood that the Green function describes the evolution of the system over one laser cycle, and the long-term evolution is obtained by applying it in successive steps.

Finally, let us emphasize that the Green function derived here does not contain the deficiencies inherent to the rescattering model [3,4], discussed in the previous contribution [2].

References

[1] L. D. Landau, E. M. Lifshic, Quantum mechanics, vol. 3, Pergamon Press, 1965.

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