A Supernova Equation of State with Light and Heavy Clusters

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Motivation I

- **equation of state** (EoS) of dense matter:

  essential ingredient in many astrophysical models
  (supernovae calculations, compact star models, . . .)

  ⇒ static properties and dynamical evolution

  ⇒ energetics, temperature, composition, . . .

- properties required for large range of **conditions**
  - density: $10^{-8} \lesssim n/n_{\rm sat} \lesssim 10$
  - temperature: $0 \leq T \lesssim 100$ MeV
  - proton fraction: $0 \leq Y_p \lesssim 0.6$

    (or isospin asymmetry $\beta = 1 - 2Y_p$)

  ⇒ **global approach needed**

Motivation II

- many EoS developed in the past:

  from simple parametizations to sophisticated models

- many investigations of detailed aspects:

  often restricted to particular conditions

- only few EoS in practical use: e.g.
  - J.M. Lattimer, F.D. Swesty
    (Nucl. Phys. A 535 (1991) 331)
  - H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi
    (Prog. Theor. Phys. 100 (1998) 1013)

- several deficiencies in applied EoS

Motivation III

aim: development of improved EoS for astrophysics

- more microscopic, self-consistent description
- well adjusted model parameters, better constrained
- relativistic approach
- additional particle species/clusters, respecting Bose/Fermi statistics
- cover widest possible range of n, T, Y_p

with special attention to

- consistency
- constraints

Outline

- General Considerations
- Relativistic Mean-Field (RMF) Model
- Light Clusters
- Generalized RMF Model
- Phase Transitions
- Heavy Clusters
- Summary and Outlook

work in progress . . . no final results

General Considerations I

- **ingredients**:

  standard: protons, neutrons

  partly optional: electrons

  optional: muons, photons, neutrinos

  - low densities: light nuclei/clusters, heavy nuclei/clusters
  - high densities: hyperons, other exotica? (pions, kaons, quarks, . . .)

- **calculation**: two-step procedure

  local minimization of free energy density $f(n, Y_p, T)$
  for given conditions n, Y_p, T (chem. equilibrium, charge neutrality, . . .)

  construction of global convexity of free energy density f
  $\Rightarrow$ phase transitions

- **results**:

  composition of matter, thermodynamical quantities

General Considerations II

- **non-hadronic** contributions
  - homogeneous spatial distribution for uncharged particles
  - only Coulomb interaction for charged particles
  - simple thermodynamics
- **hadronic** contributions
  - possible general strategy:

    start with given nucleon-nucleon interaction,
    apply machinery of many-body theory . . . (cf. Dirac-Brueckner calculations)
    . . . too complicated, too limited
  - here: more phenomenological approach, combination
    - **relativistic mean-field model** with density-dependent nucleon-meson couplings
    - **generalized Beth-Uhlenbeck approach** (⇒ light clusters)
    - **relativistic Thomas-Fermi approximation** (⇒ heavy clusters)

Relativistic Mean-Field (RMF) Model I

- standard **Lagrangian density** of Walecka type
  with nucleons (ψ), mesons ($\sigma, \omega_\mu, \vec{\rho}_\mu$) and photons ($A_\mu$) as degrees of freedom
  - mesons: convenient auxiliary fields, same quantum numbers as "real" mesons
  - only minimal (linear) meson-nucleon couplings
  - density-dependent meson-nucleon couplings Γ_i
    - functional form as suggested by Dirac-Brueckner calculations of nuclear matter
    - more flexible approach than models with non-linear meson self-interactions
  - parameters: nucleon/meson masses, coupling strengths/density dependence
    - in total **8 - 9 free parameters** (highly correlated)
    - fitted to properties of finite nuclei
      (binding energies, spin-orbit splittings, charge/diffraction radii, surface/neutron skin thicknesses)

$\Rightarrow$ nucleon/meson/photon field equations, solved in mean-field approximation
(Hartree approximation, no-sea approximation, classical meson/photon fields)

RMF Model II - Couplings

- **nucleon-meson couplings** $\Gamma_i = \Gamma_i(\varrho)$

  functionals of vector density $\varrho = \sqrt{j_\mu j^\mu}$ with $j_\mu = \bar{\psi}\gamma_\mu\psi$

  ◦ rational/exponential functions

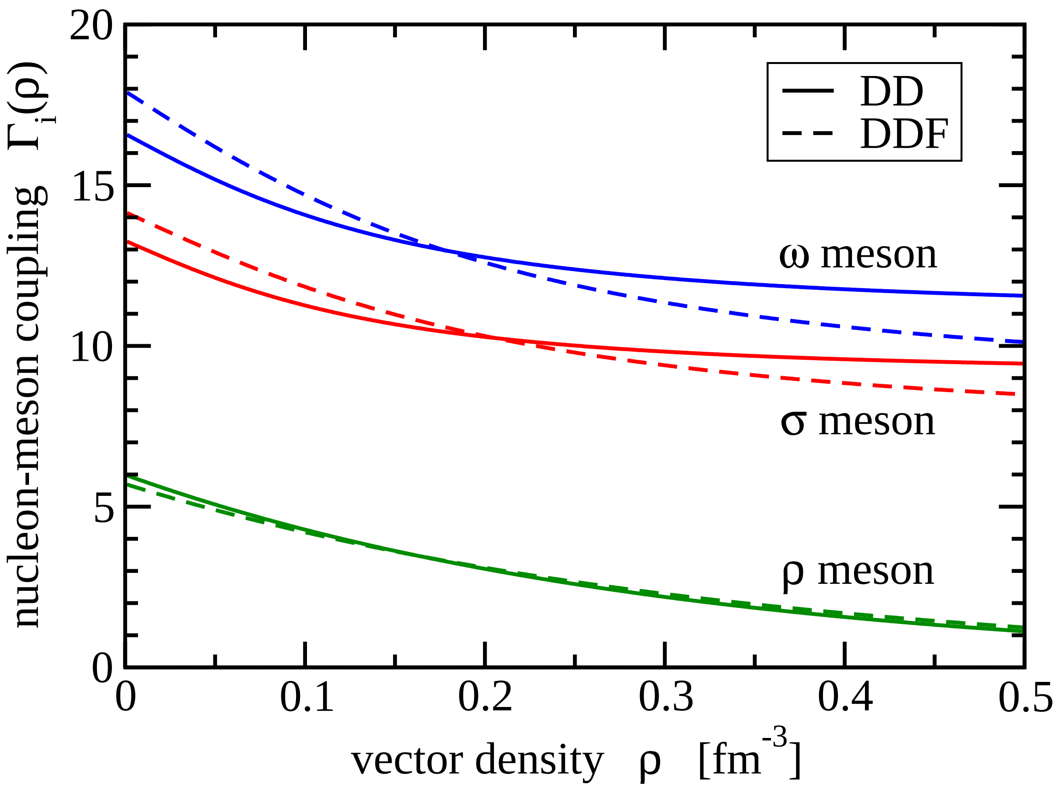
  ◦ well controlled asymptotics

  ⇒ rearrangement contributions
  in nucleon self-energies

  ⇒ thermodynamical consistency

  $p_{\text{field}} = \frac{1}{3}\sum_{m=1}^{3}\langle T^{mm}\rangle$

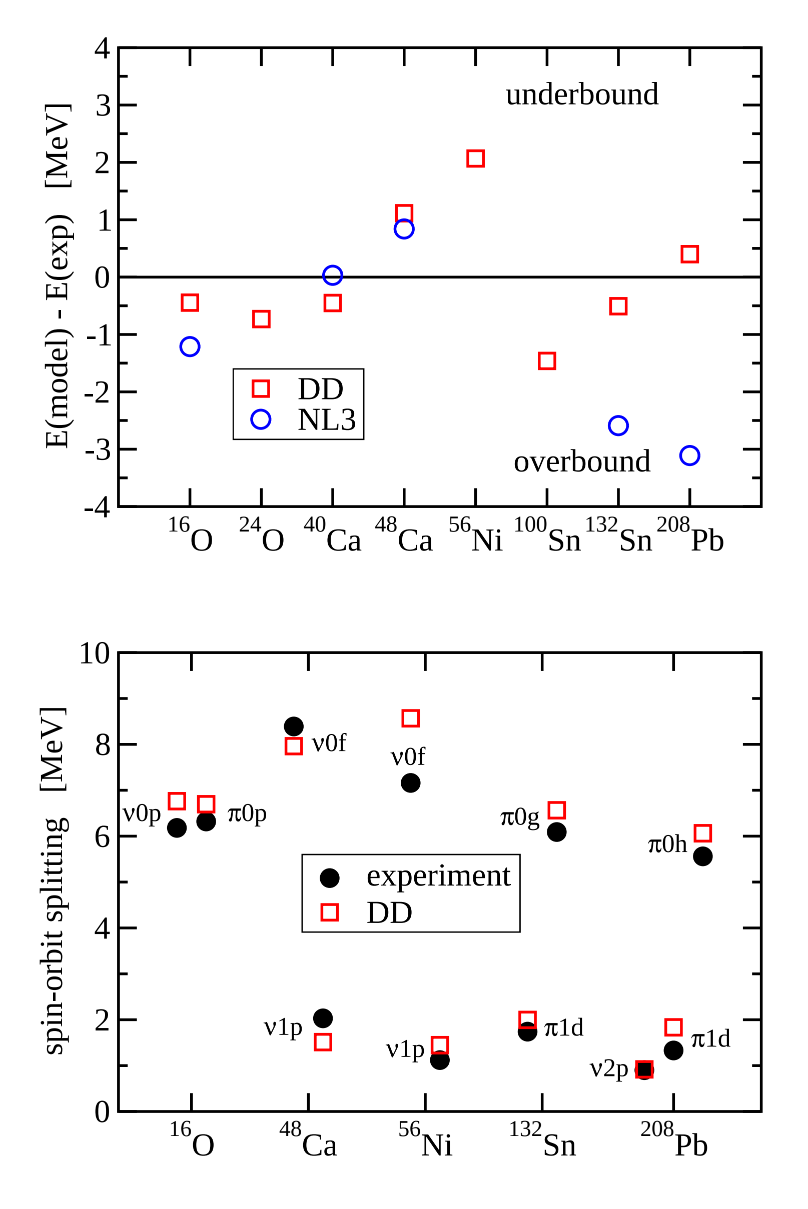
  $= p_{\text{thermo}} = n^2 \left.\frac{\partial(\varepsilon/n)}{\partial n}\right|_{T,V}$

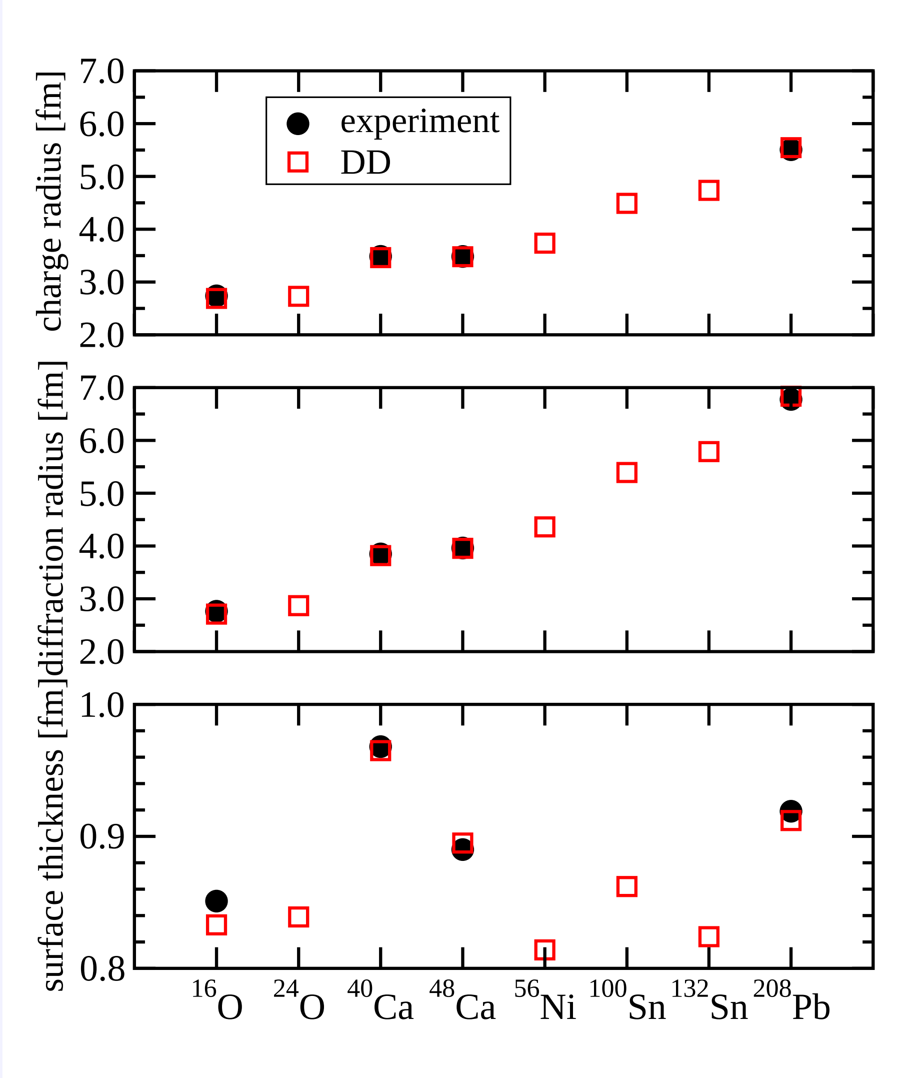


S. Typel, H.H. Wolter, Nucl. Phys. A 656 (1999) 331;

C. Fuchs, H. Lenske, H.H. Wolter, Phys. Rev. C 52 (1995) 3043

RMF Model III - Properties of Nuclei





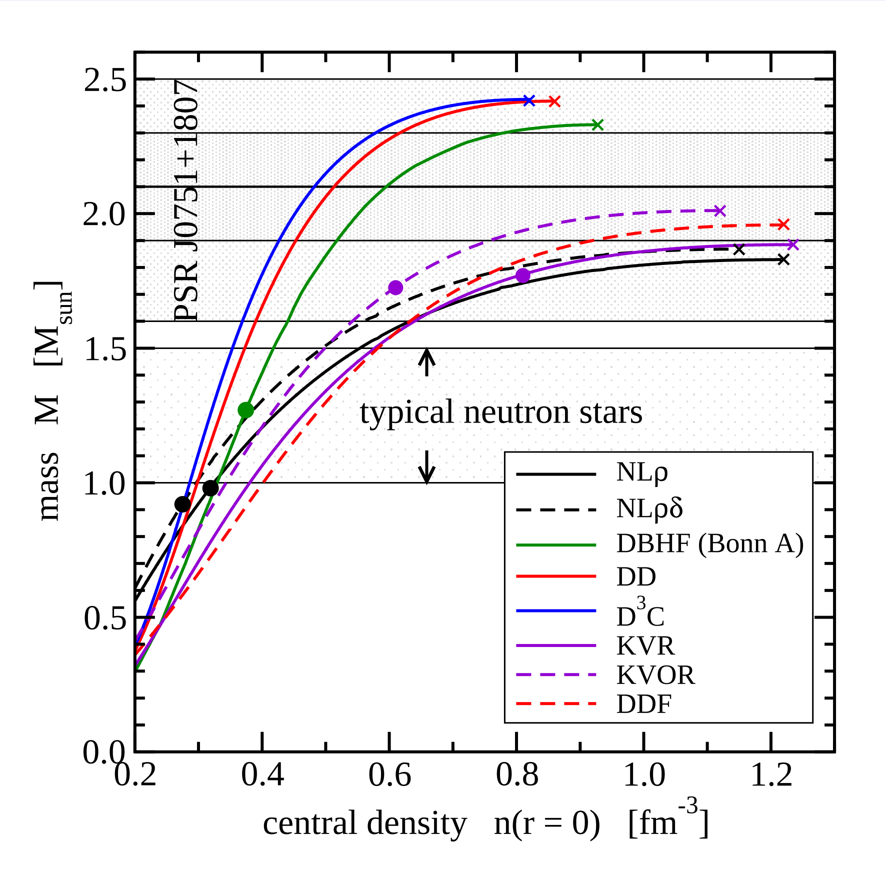
DD: S. Typel, Phys. Rev. C 71 (2005) 064301

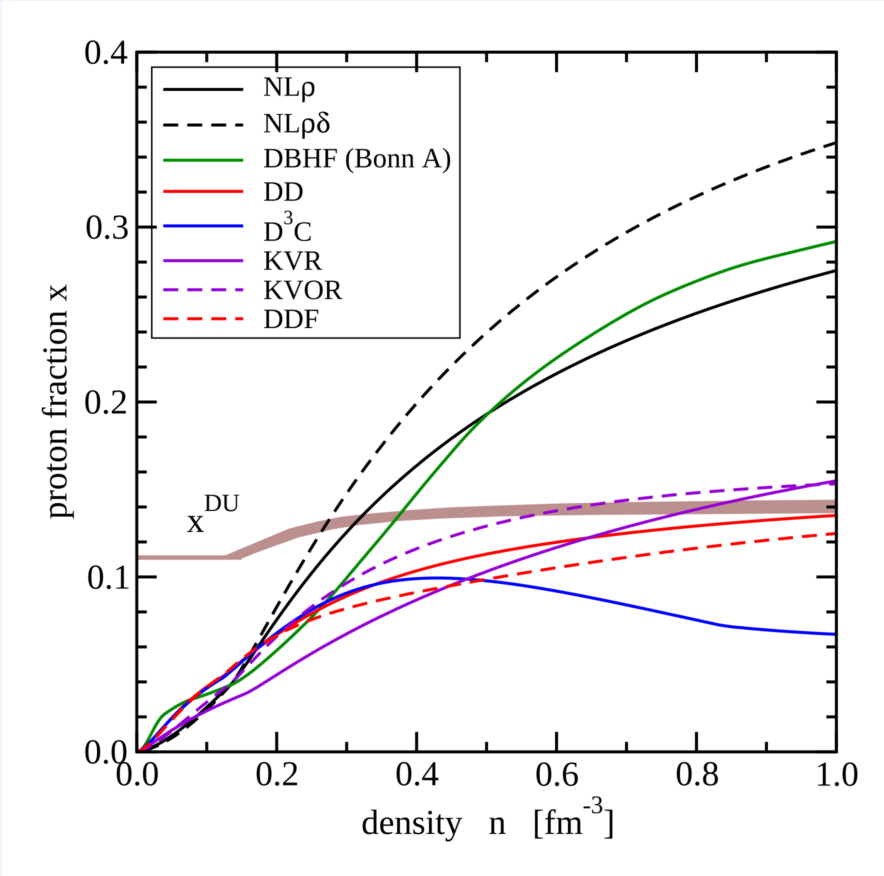
NL3: G. A. Lalazissis, J. König, P. Ring, Phys. Rev. C 55 (1997) 540

RMF Model IV - Constraints from Astronomy

- stiffness of symmetric matter EoS
  ⇒ maximum mass of neutron star

- stiffness of symmetry energy E_s
  ⇒ direct URCA process, cooling



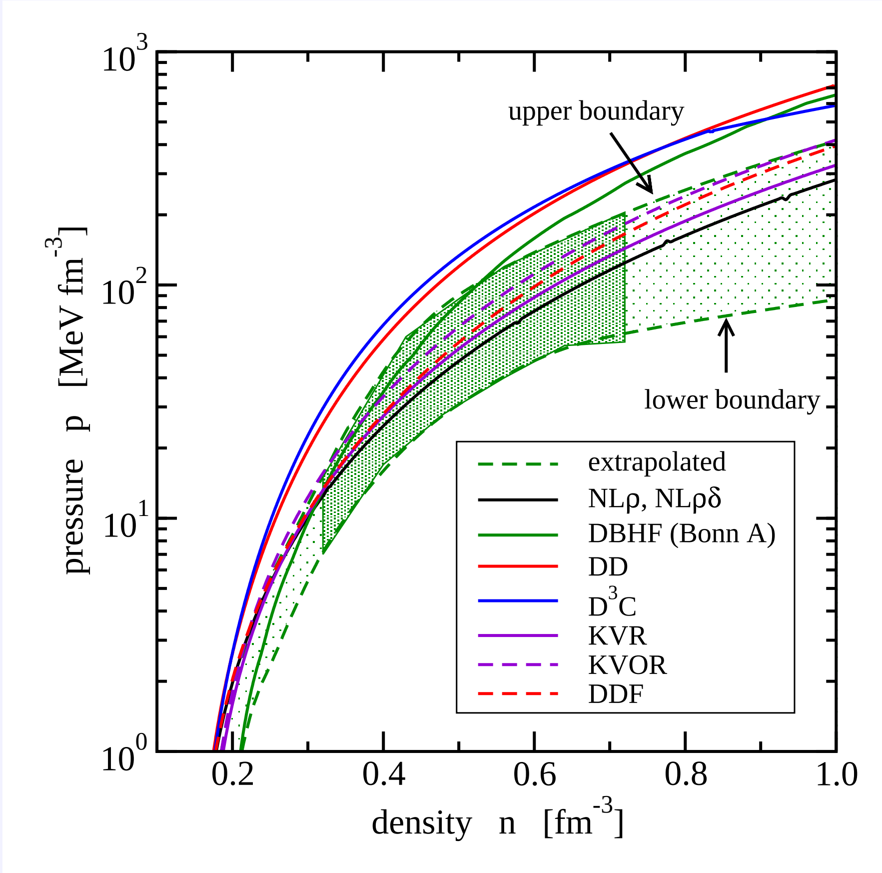


additional constraints: • mass-radius relation • gravitational mass-baryon number relation

(T. Klähn et al., Phys. Rev. C 74 (2006) 035802)

RMF Model V - Constraints from Heavy-Ion Collisions

- elliptic flow in heavy-ion collisions
  ⇒ test of symmetric part of EoS
  at high densities
  - model analysis of experimental data
    ⇒ allowed region in pressure-density diagram
    ⇒ constraint on stiffness of EoS
    (P. Danielewicz et al., Science 298 (2002) 1592)
  - but: conclusions ambiguous,
    new degrees of freedom at high energies,
    momentum dependence of interaction,
    model dependence of analysis



additional constraints:

- sub-threshold kaon production
  ⇒ soft EoS of symmetric nuclear matter (C. Fuchs, Prog. Part. Nucl. Phys. 56 (2006) 1)
- isoscaling in fragment isotopic yields
  ⇒ soft symmetry energy ($L \approx 60 - 80$ MeV) (D.V. Shetty et al., PRC 75 (2007) 034603)

Light Clusters I

low densities:

two-, three-, . . . many-body correlations due to NN interaction ⇒

- ∘ modification of thermodynamical properties
- ∘ bound states appear as new particle species

theoretical approaches:

- virial expansion in classical description (with potentials)
  - ∘ expansion of grand-canonical partition function $\mathcal{Z}(V, T, \mu_i)$ in powers of fugacities $z_i = \exp(\mu_i/T) \ll 1$ with chemical potentials μ_i
- quantum mechanical generalization (with density of states)
  (G. E. Beth and E. Uhlenbeck Physica 3 (1936) 729, Physica 4 (1937) 915)
- medium effects included in thermodynamical Green's function appoach
  (M. Schmidt, G. Röpke and H. Schulz, Ann. Phys. 202 (1990) 57)

Light Clusters II - Virial Expansion/BU Approach

- second quantummechanical virial coefficient $\Leftrightarrow$ correlation due to interaction

$$b_{ij}(T) = \frac{1+\delta_{ij}}{2}\frac{\lambda_i^{3/2}\lambda_j^{3/2}}{\lambda_{ij}^3}\int dE\, D_{ij}(E)\exp\left(-\frac{E}{T}\right) \qquad \lambda_i = \hbar\sqrt{\frac{2\pi}{m_i T}}$$

with density of states
$$D_{ij}(E) = \sum_k g_k^{(ij)}\delta(E - E_k^{(ij)}) + \sum_l g_l^{(ij)}\frac{1}{\pi}\frac{d\delta_l^{(ij)}}{dE}$$

  - contributions from bound states at energies $E_k^{(ij)} < 0$ (degeneracy $g_k^{(ij)}$)
  - contributions from continuum states in channels l with phase shifts $\delta_l^{(ij)}(E)$
  - corrections for Fermi or Bose statistics

- if bound state energies $E_k^{(ij)} < 0$ and phase shifts $\delta_l^{(ij)}$ are known experimentally

  $\Rightarrow$ low-density behaviour of EoS established model-independently

  (e.g. C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

- natural upper limit for virial expansion: $n_i\lambda_i^3 < 1$

Light Clusters III - Cluster Formation

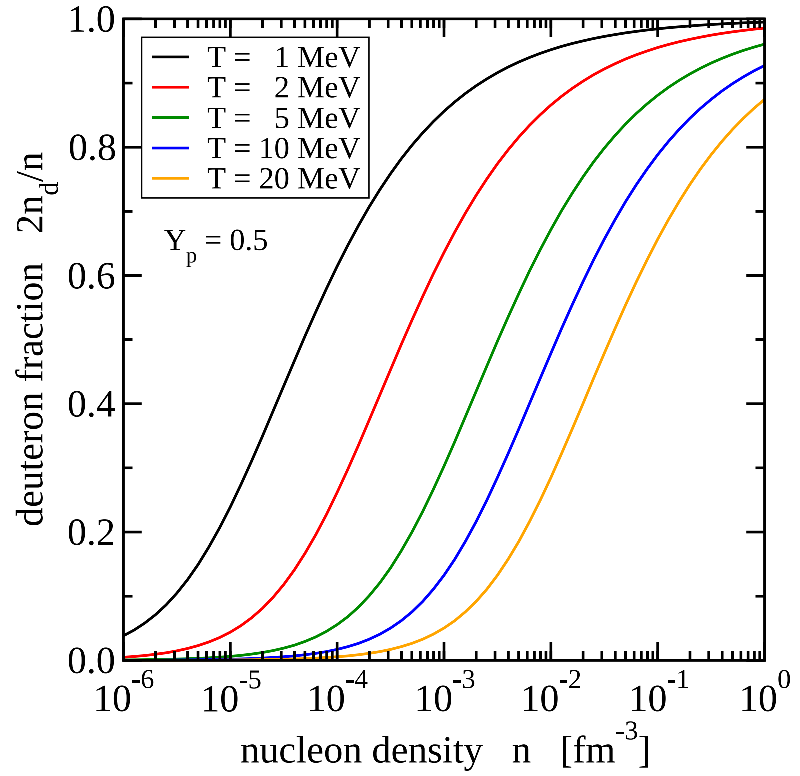
example

- deuteron formation in symmetric nuclear matter:

  total nucleon density $n = n_{\text{free}} + 2n_d$ with

  $$n_d = 3 \cdot 2^{-\frac{5}{2}} \lambda^3 n_{\text{free}}^2 \exp(-E_d/T)$$

  $\Rightarrow$ law of mass action $n + p \Leftrightarrow d$



general case

- nuclear statistical equilibrium (NSE)
- high densities: unphysical behaviour,
  no clusters but homogeneous matter expected

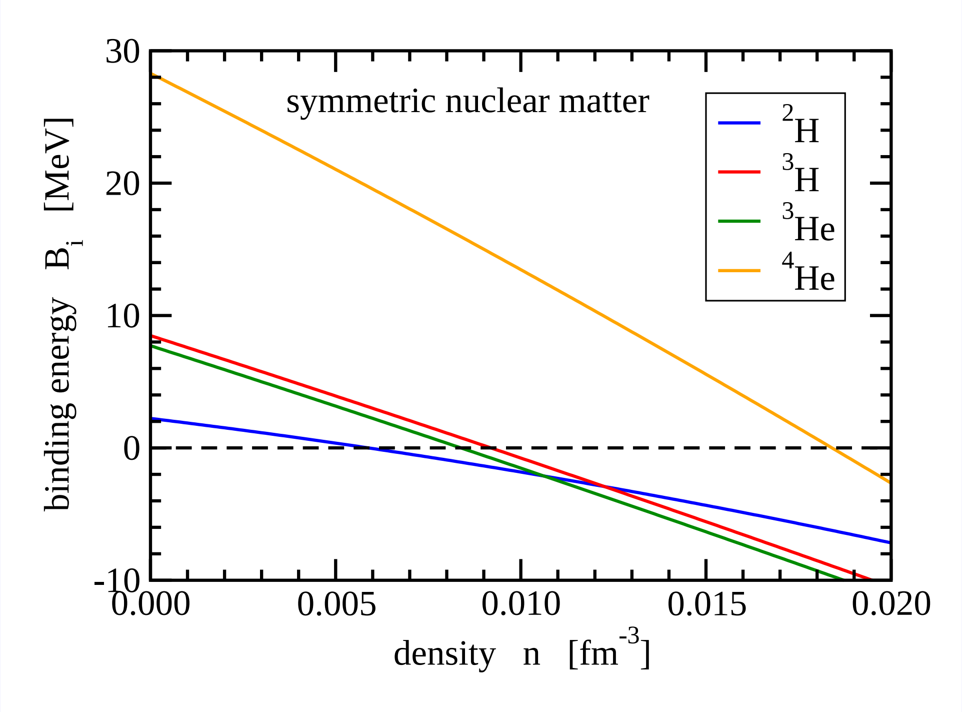
$\Rightarrow$ generalization necessary:

- ○ Pauli blocking of states
- ○ in-medium interaction
- ○ modification of cluster properties
- ○ three-body (^{3}H, ^{3}He), four-body (^{4}He) correlations and beyond
- ○ Boson/Fermion statistics

Light Clusters IV - Generalized BU Approach

- thermodynamic Green's function approach
  (M. Schmidt, G. Röpke, H. Schulz, Ann. Phys. 202 (1990) 57)
  ⇒ go beyond quasi-particle approximation of spectral function
  ⇒ generalized density of states with
    - medium-dependent shift of of binding energies $E_k = E_k(P, \mu, T)$,
    - generalized scattering phase shifts $\delta_l = \delta_l(E, P, \mu, T)$ from in-medium T-matrix

  ⇒ model calculations needed
  (G. Röpke, arXiv:0810.4645)



- combine generalized Beth-Uhlenbeck approach with relativistic mean-field model

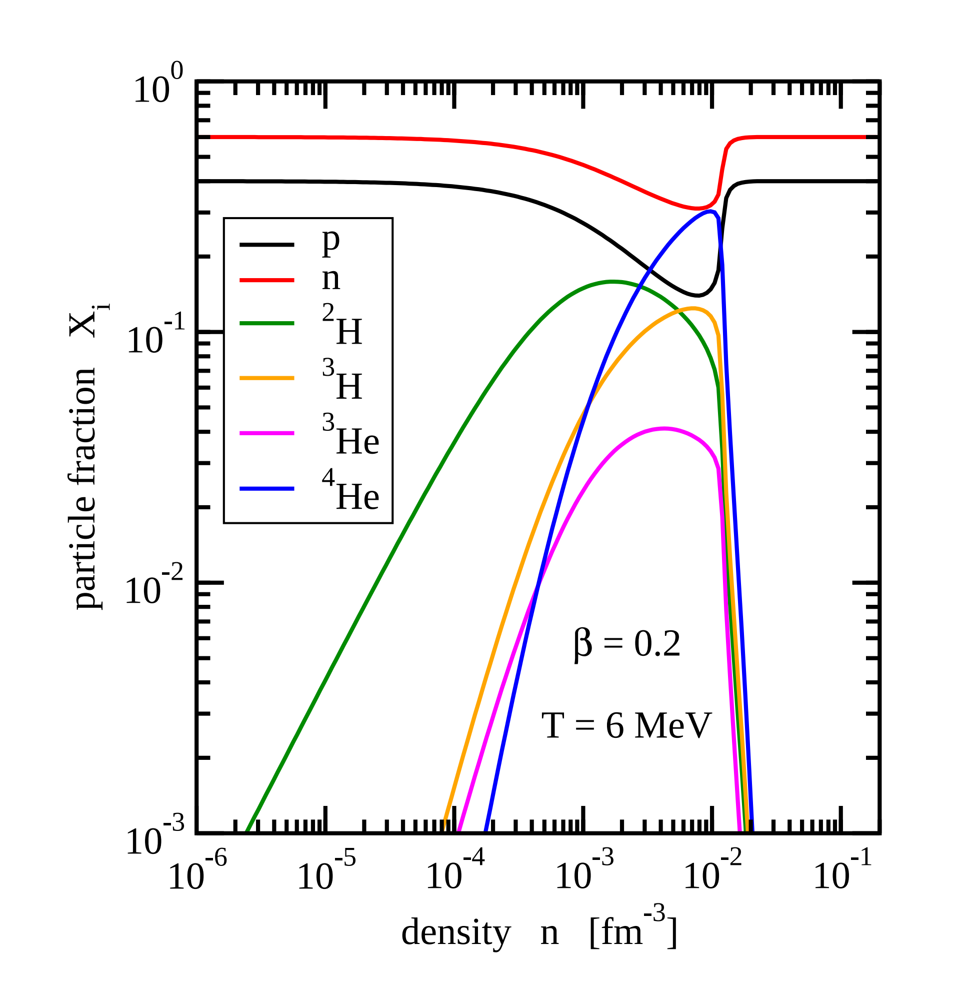
  ⇒ treat cluster states as explicit degrees of freedom in RMF Lagrangian

  ⇒ generalized RMF model

Generalized RMF model I - Light Clusters

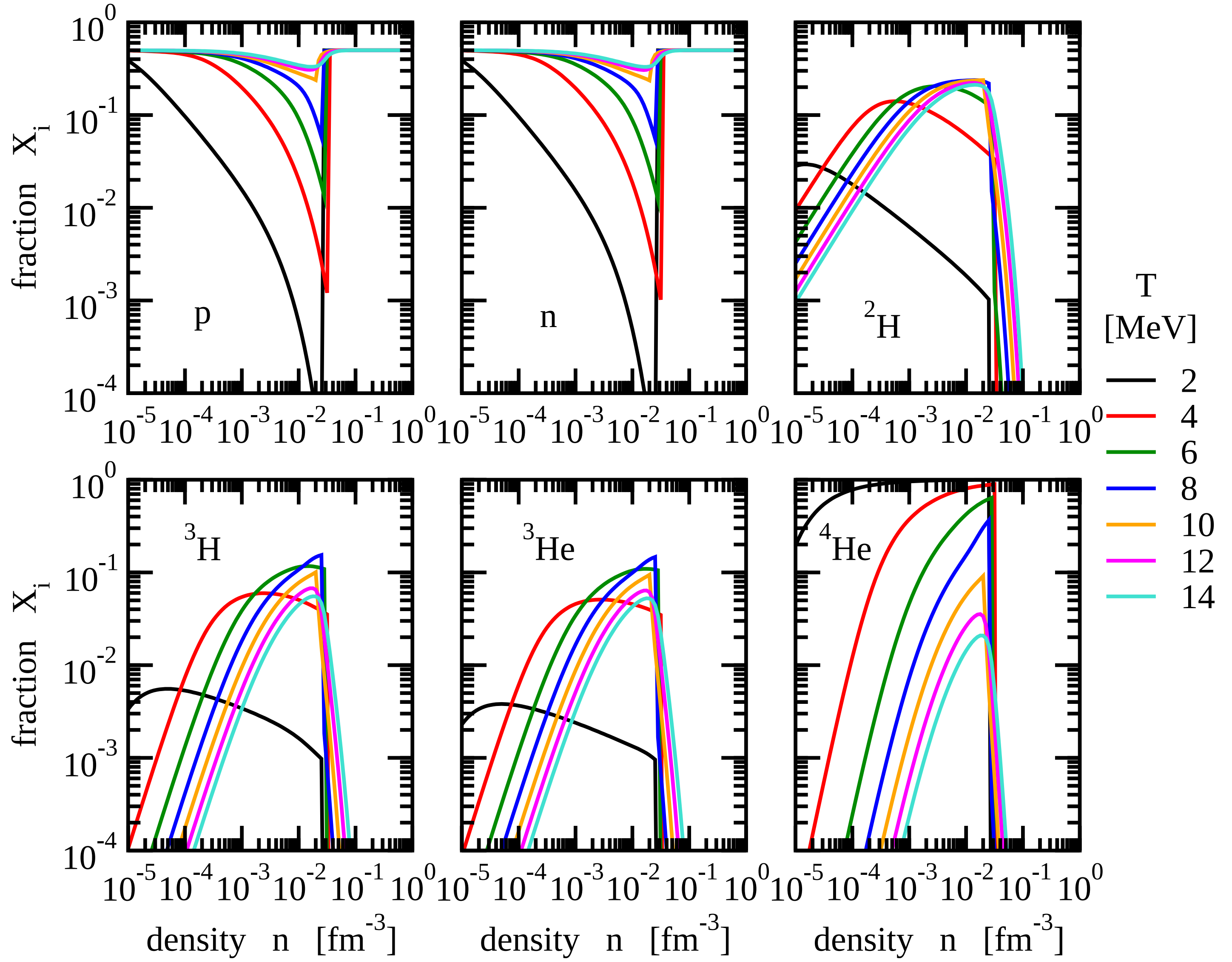
- consider 2-, 3-, and 4-body correlations in the medium
  - presently only bound states (deuterons, tritons, helions, and alphas)
  - density and temperature dependence of effective binding energies

    ⇒ new 'rearrangement' contributions to self-energies, entropy density, . . . essential for thermodynamical consistency
- Mott effect: clusters dissolve at high densities
- correct limits for low and high densities
- no heavy clusters/phase transition included here



Generalized RMF Model II - Particle Fractions

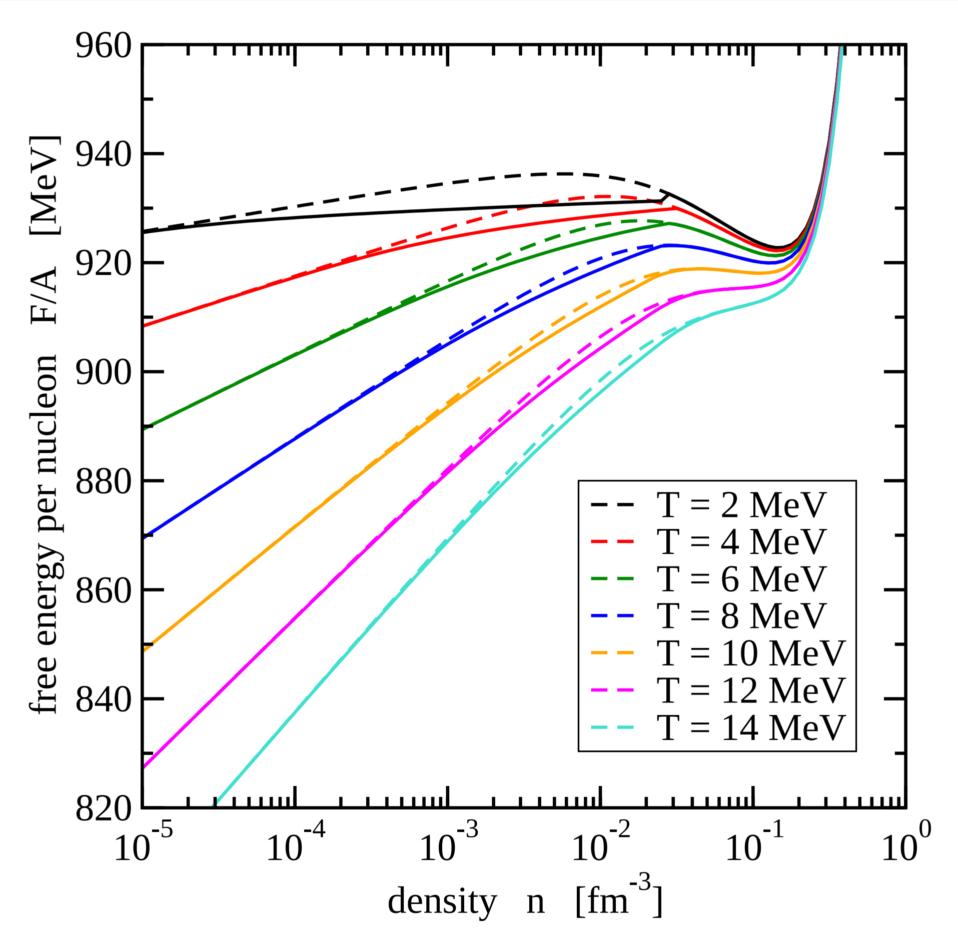
$X_i = A_i \frac{n_i}{n}$ for symmetric nuclear matter ($\beta = 0.0$)



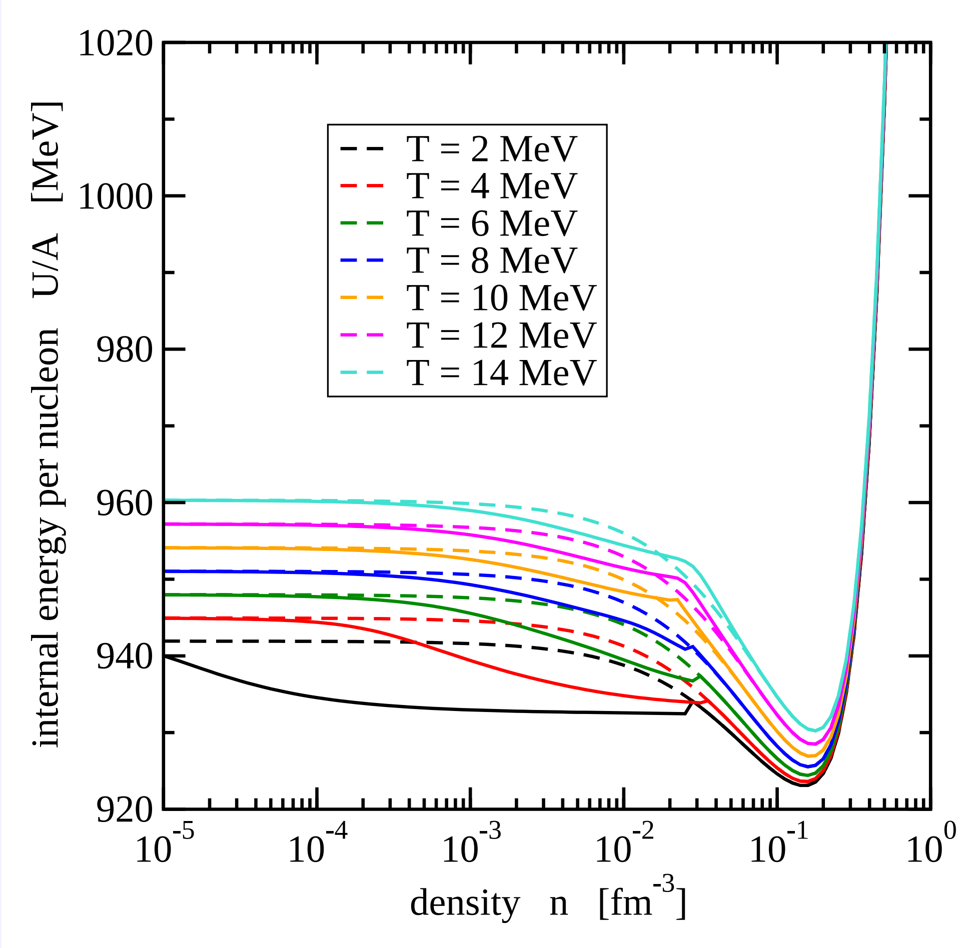
Generalized RMF Model III - Energies

without (dashed) and with (solid) clusters for symmetric nuclear matter ($\beta = 0.0$)

free energy per nucleon $F/A = f/n$



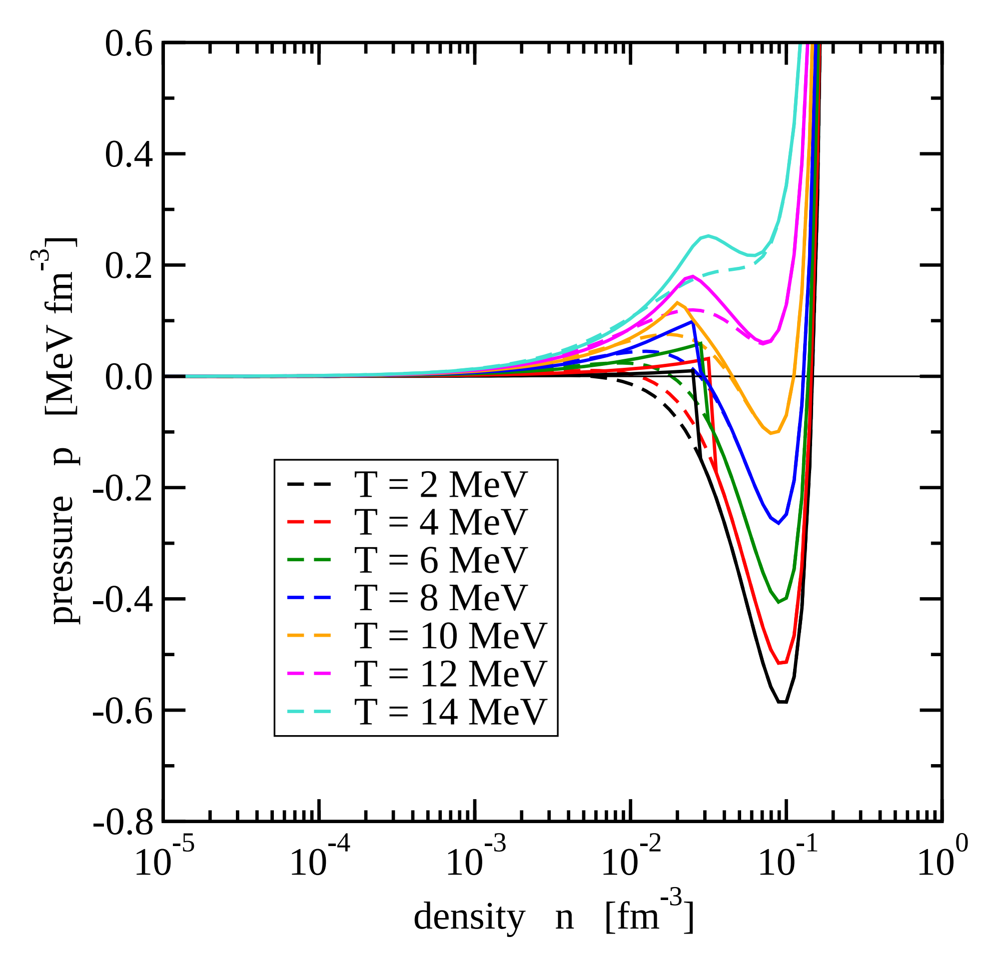
internal energy per nucleon $U/A = u/n$



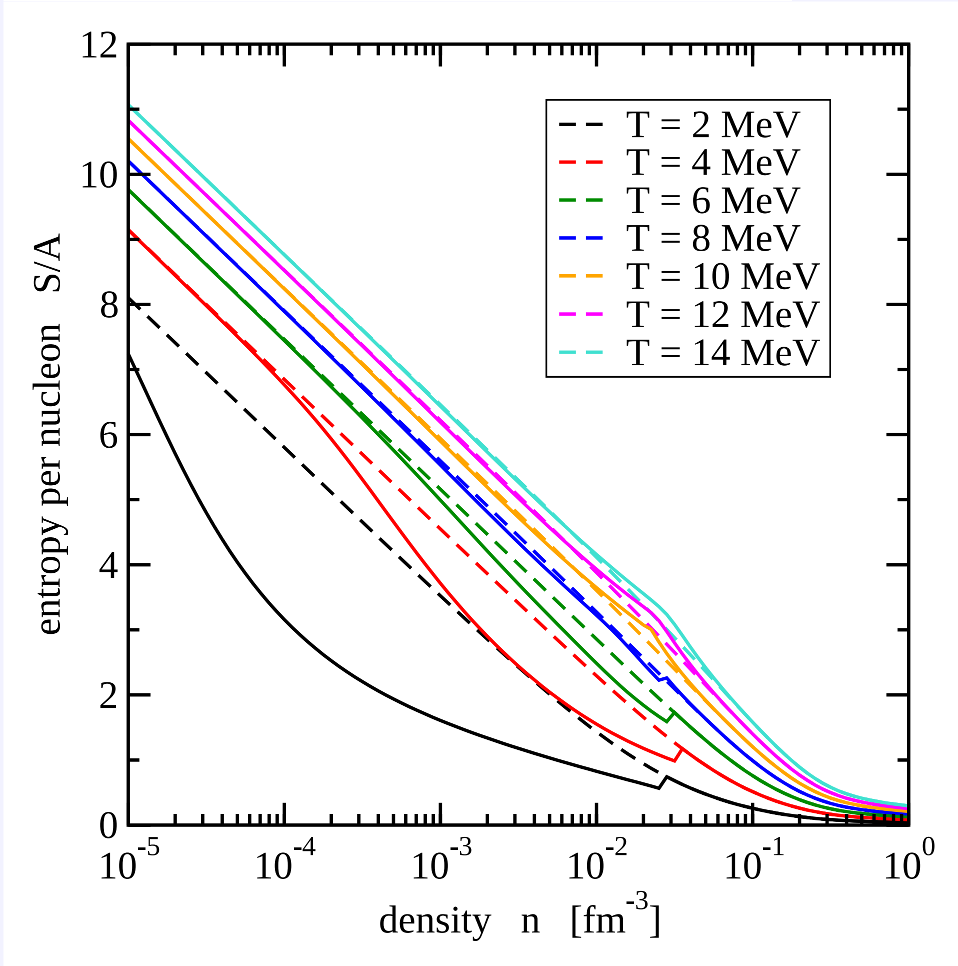
Generalized RMF Model IV - Pressure and Entropy

without (dashed) and with (solid) clusters for symmetric nuclear matter ($\beta = 0.0$)

pressure p



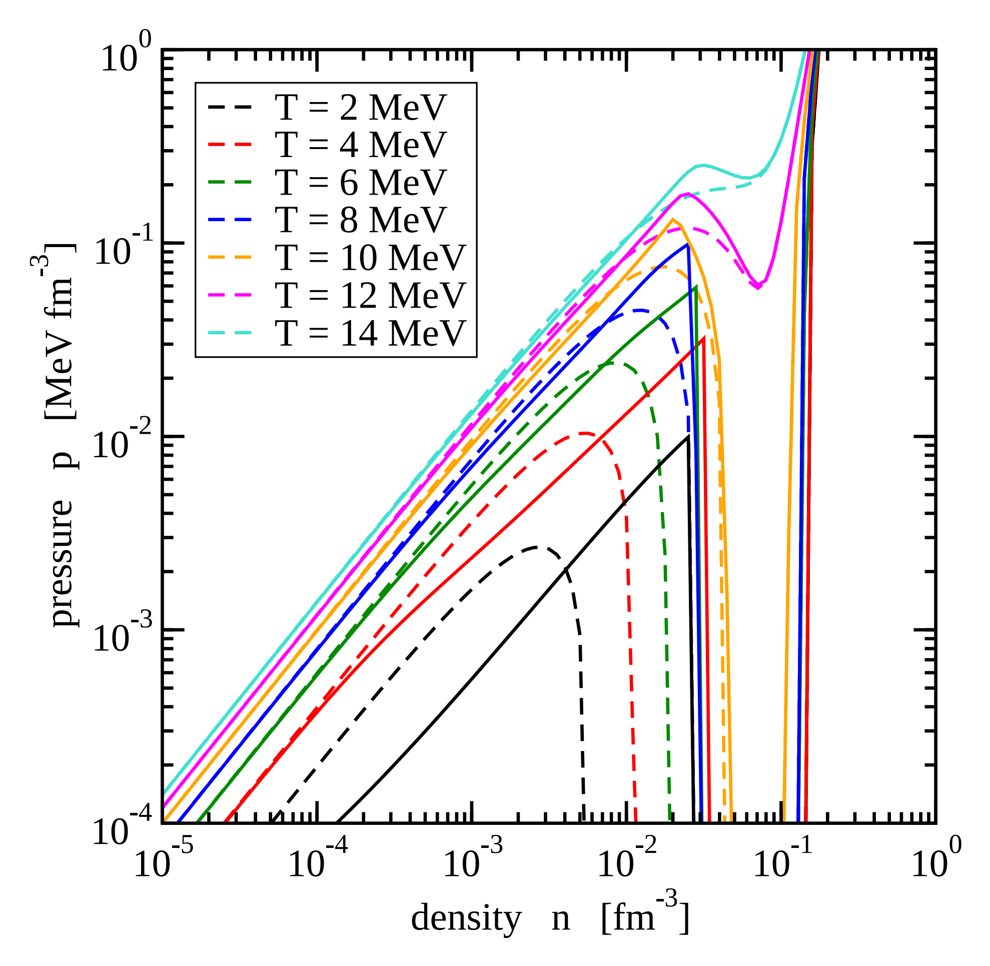
entropy per nucleon $S/A = s/n$



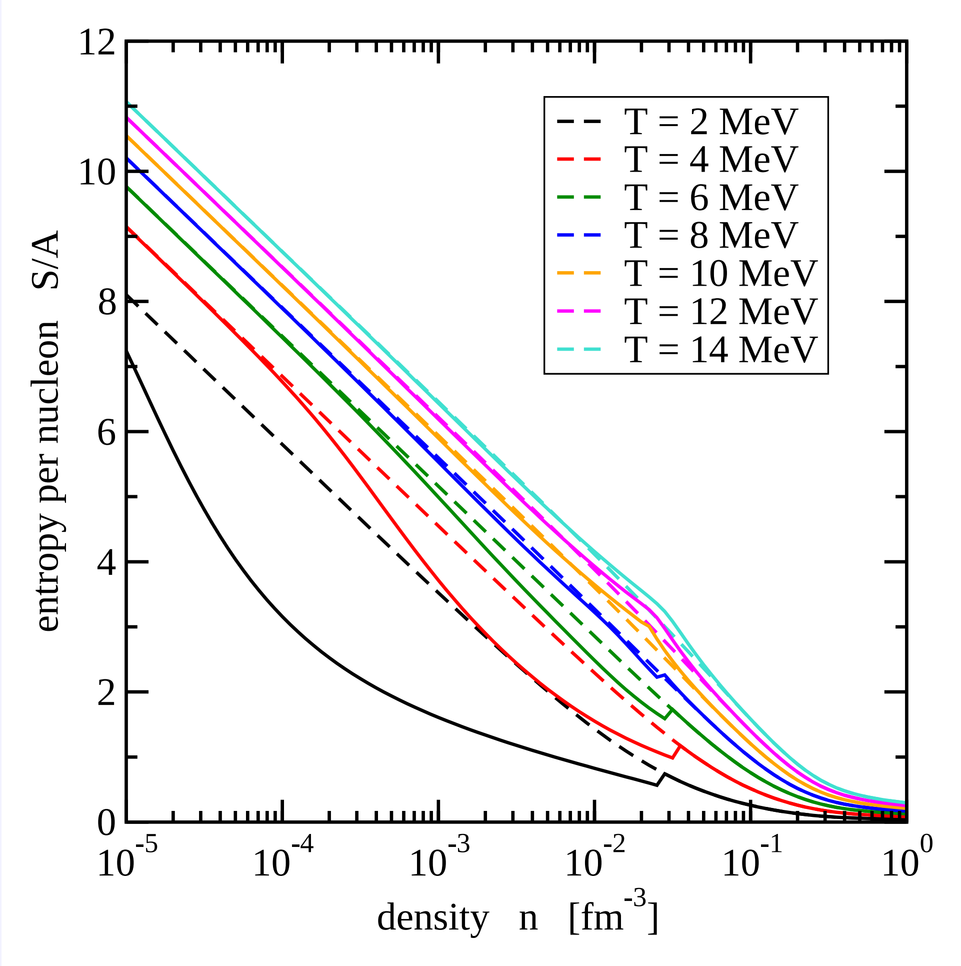
Generalized RMF Model IV - Pressure and Entropy

without (dashed) and with (solid) clusters for symmetric nuclear matter ($\beta = 0.0$)

pressure p

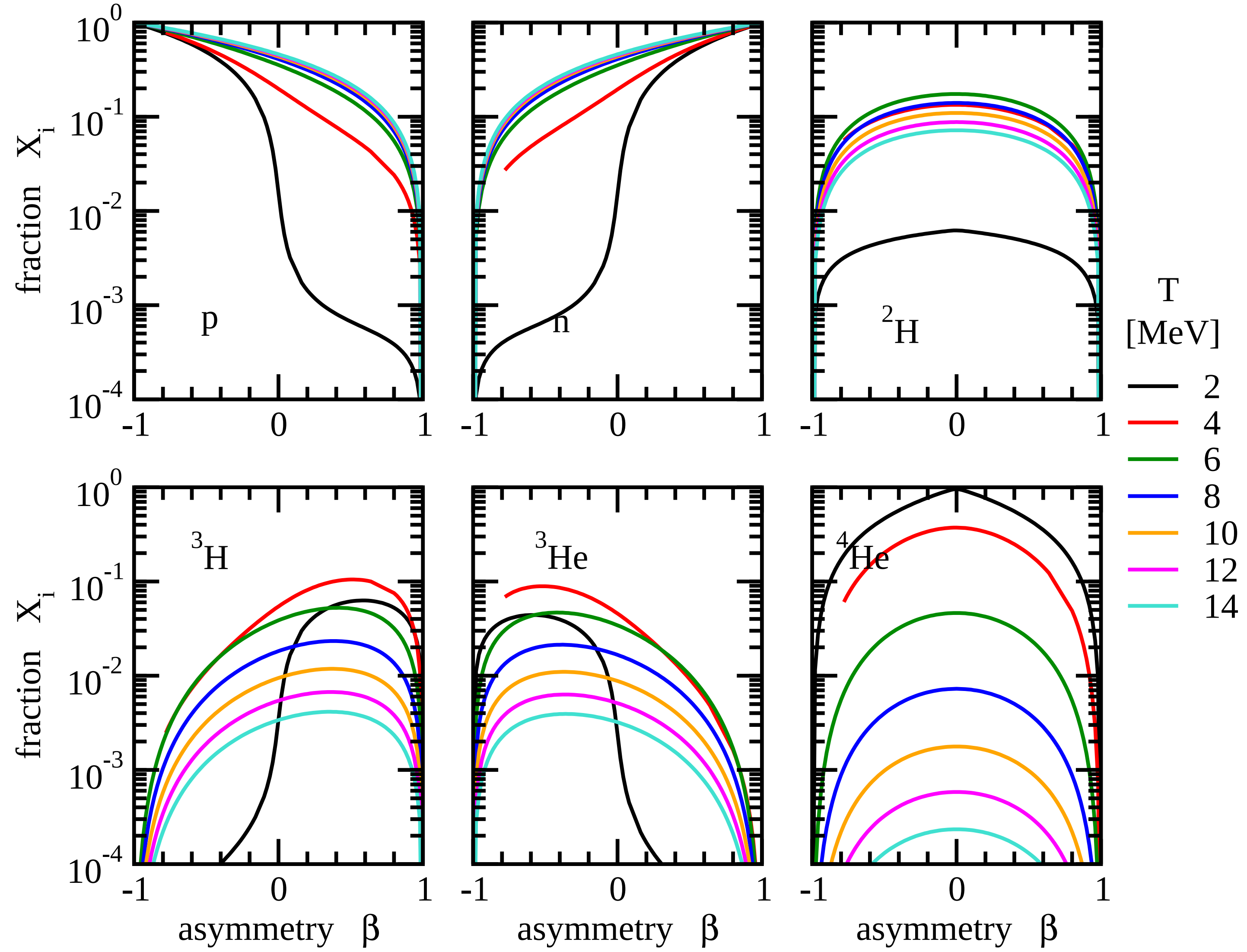


entropy per nucleon $S/A = s/n$



Generalized RMF Model V - Particle Fractions

$X_i = A_i \frac{n_i}{n}$ for density $n = 10^{-3}$ fm^{-3}



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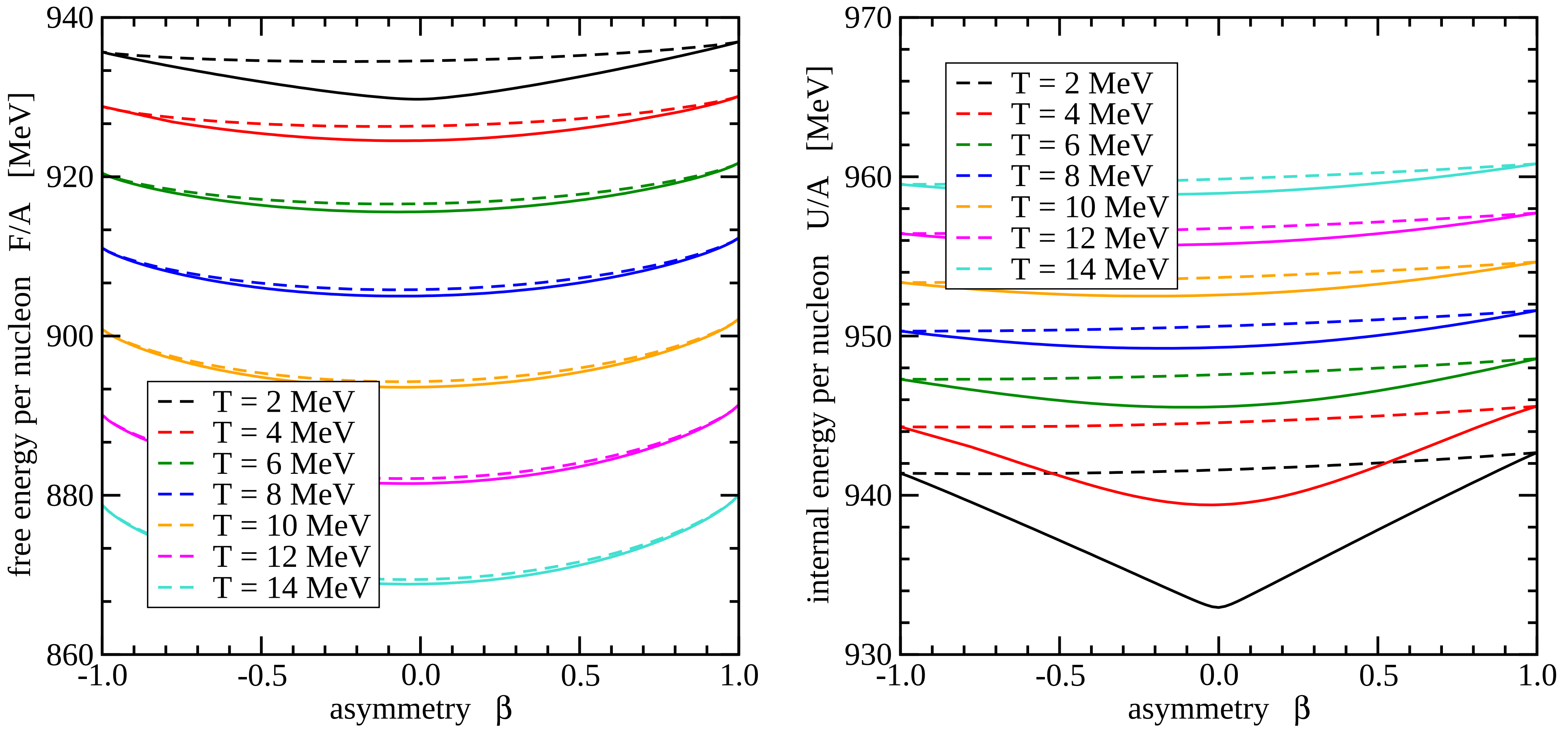
Generalized RMF Model VI - Energies

without (dashed) and with (solid) clusters for density $n = 10^{-3}$ fm^{-3}

free energy per nucleon $F/A = f/n$

internal energy per nucleon $U/A = u/n$



Generalized RMF Model VII - Symmetry Energy

- general definition for zero temperature:

$$E_s(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n,\beta)\Big|_{\beta=0}$$

$\Rightarrow$ nuclear matter parameters

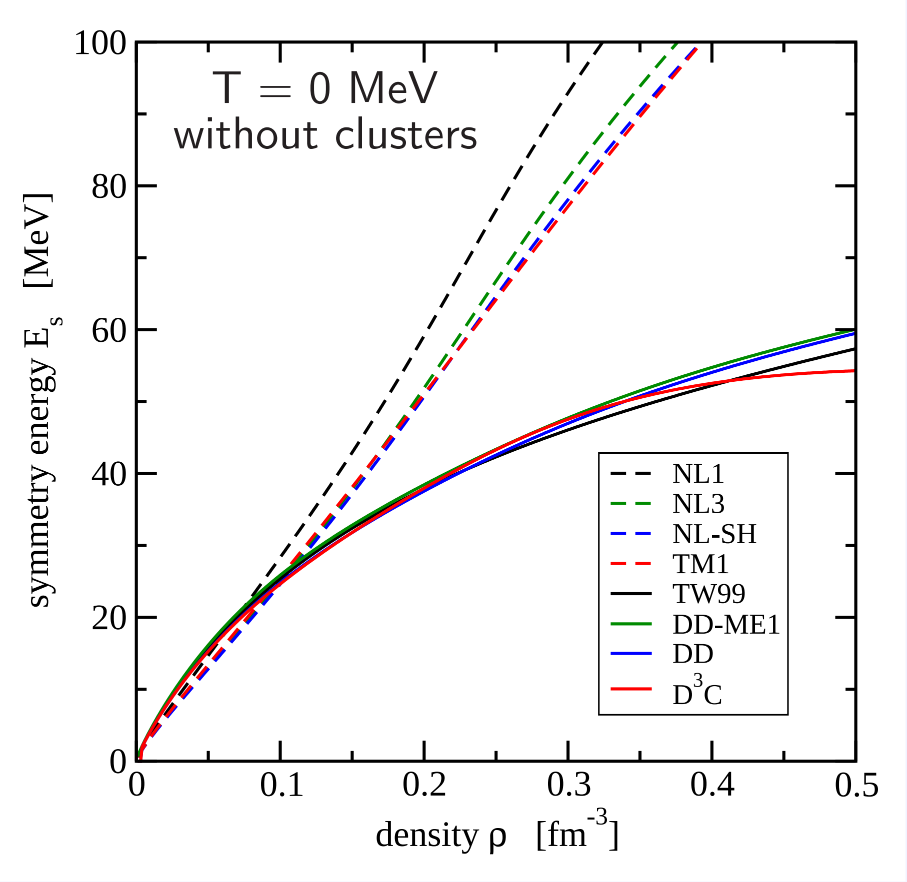
$$J = E_s(n_{\text{sat}}) \quad L = 3n\frac{d}{dn}E_s\Big|_{n=n_{\text{sat}}}$$

- correlation: neutron skin thickness

$\Leftrightarrow$ slope of neutron matter EoS ($\Leftrightarrow L$)

B. A. Brown, Phys. Rev. Lett. 85 (2000) 5296,

S. Typel, B. A. Brown, Phys. Rev. C 64 (2001) 027302



- with clusters and at finite temperatures:
  - use approximation

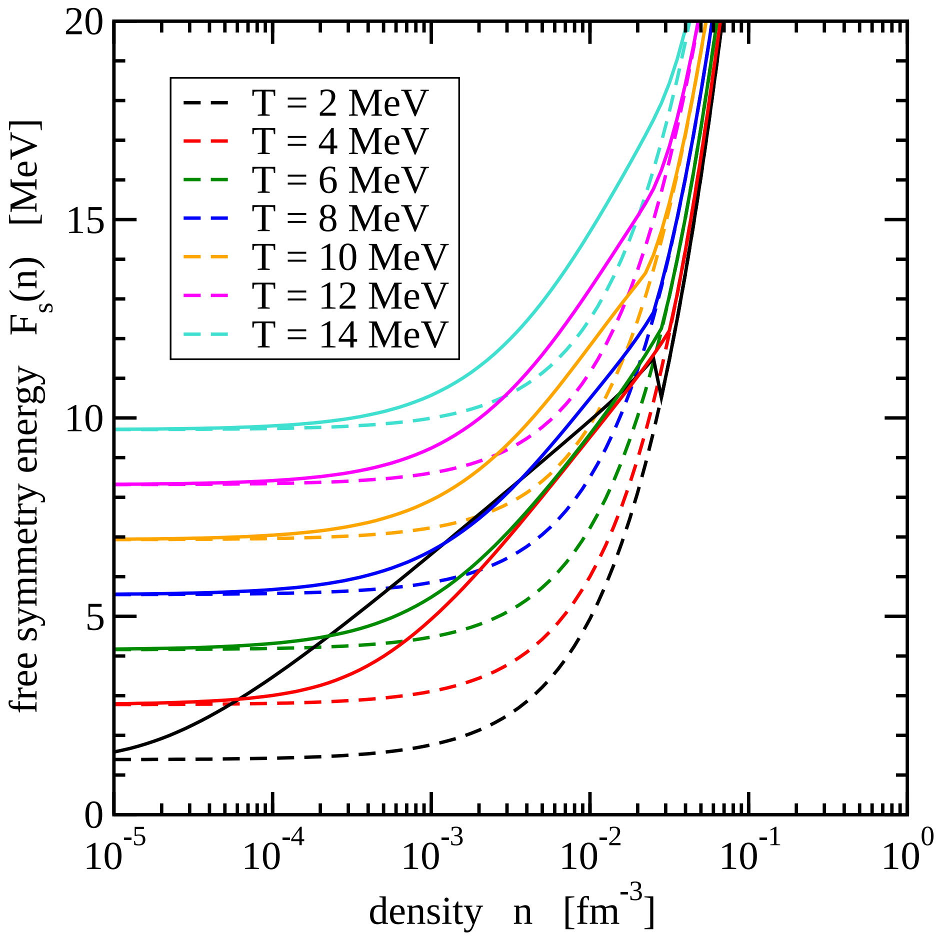
$$E_s(n) = \frac{1}{2}\left[\frac{E}{A}(n,1) - 2\frac{E}{A}(n,0) + \frac{E}{A}(n,-1)\right]$$

  - distinguish free symmetry energy F_s and internal symmetry energy U_s

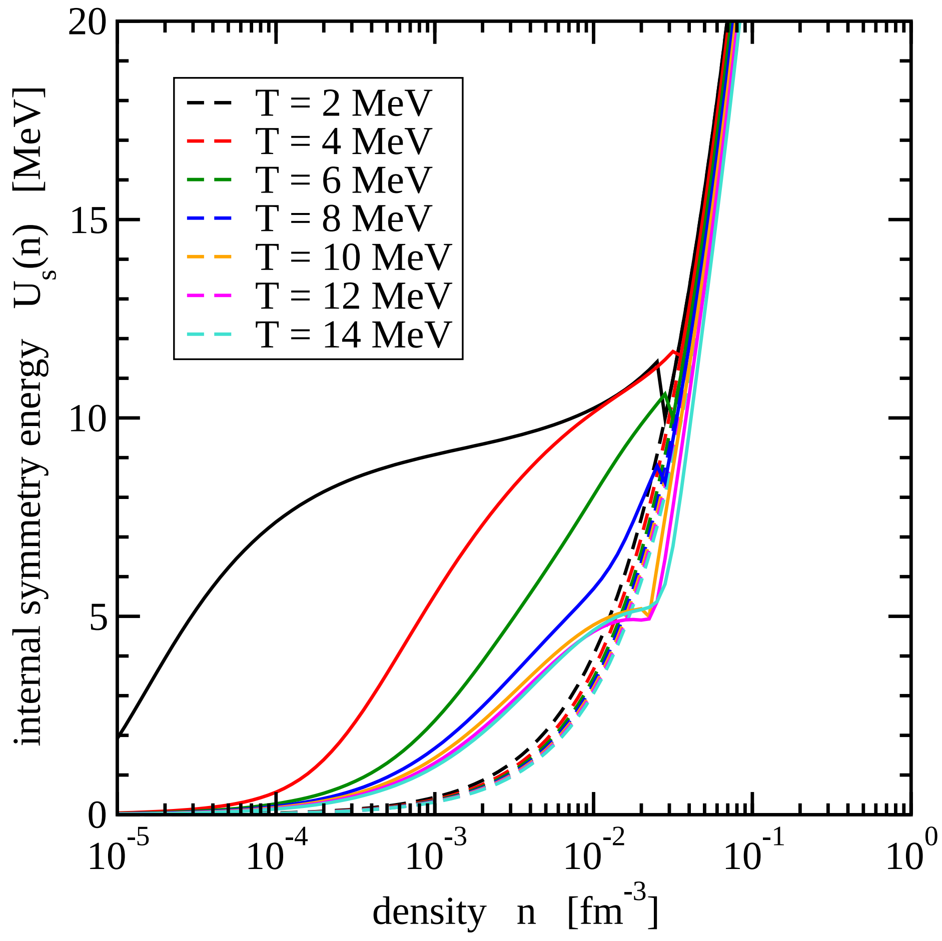
Generalized RMF Model VIII - Symmetry Energies

without (dashed) and with (solid) clusters

free symmetry energy



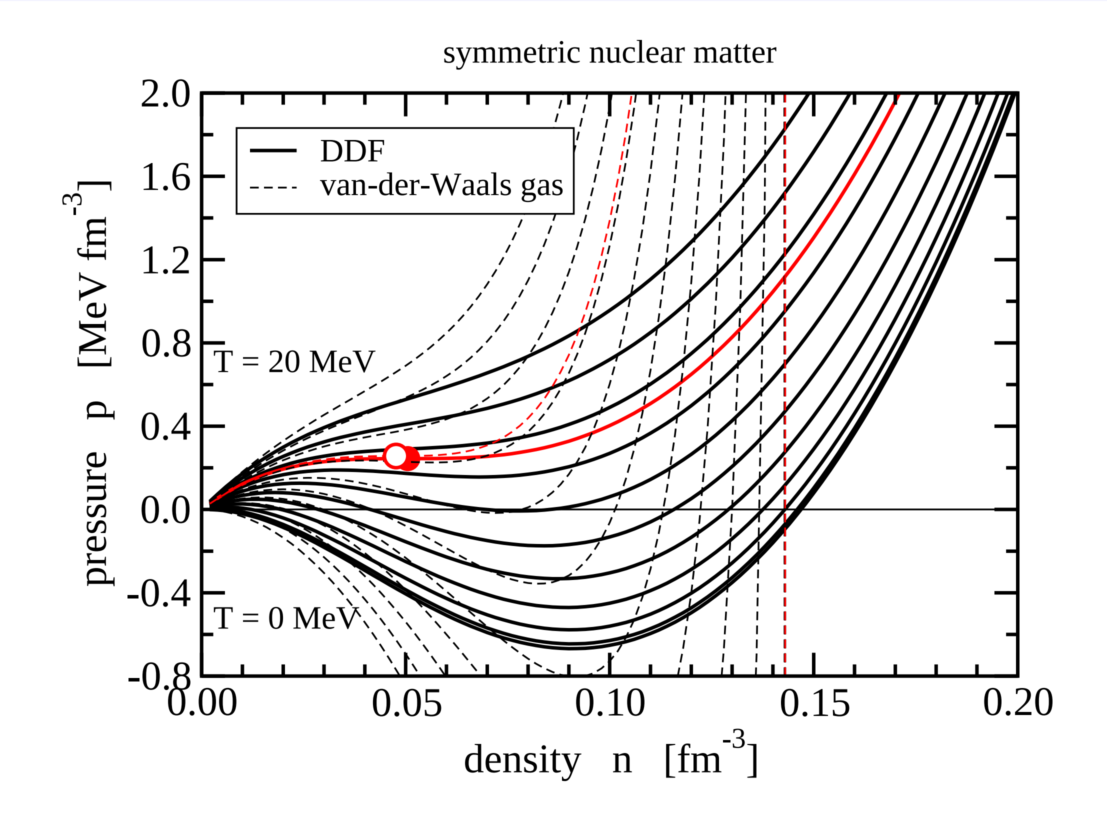
internal symmetry energy



example

- homogeneous symmetric nuclear matter without clusters
- consider isothermes in pressure-density diagram
  $\Rightarrow$ critical point

  parametrization DDF:
  $T_c = 15.2$ MeV, $n_c = 0.0505$ fm^{-3},
  $p_c = 0.244$ MeV fm^{-3}
  $\Rightarrow p_c/(n_c T_c) = 0.318$
  cf. van-der-Waals gas:
  $p_c/(n_c T_c) = 0.375$



- $T < T_c$: inhomogeneous matter (various length scales)

  $\Rightarrow$ liquid-gas phase transition (high/low densities)

  $\Rightarrow$ low temperatures and low densities: solid state (crystal, pasta phases)

Phase Transitions II

general construction for asymmetric nuclear matter

(see, e.g., H. Müller and B. D. Serot, Phys. Rev. C 52 (1995) 2072)

- phase transition with two conserved charges:
  neutron and proton number or baryon and baryonic charge number
  relation of chemical potentials: $\mu_b = \mu_n$, $\mu_q = \mu_p - \mu_n$
  $\Rightarrow$ standard Maxwell construction not applicable

- distinction necessary:
  - spinodals (instability boundaries) $\Leftrightarrow$ local criterion on free energy density f:
    stable if matrix $\left. \left(\frac{\partial^2 f}{\partial n_i \partial n_j} \right) \right|_{T,V}$ positive (e.g. mechanical and diffusive stability)
  - binodals (phase separation boundaries) $\Leftrightarrow$ global criterion on free energy density f:
    convexity of free energy density

- spinodals enclosed by binodals $\Rightarrow$ binodals relevant for system in equilibrium

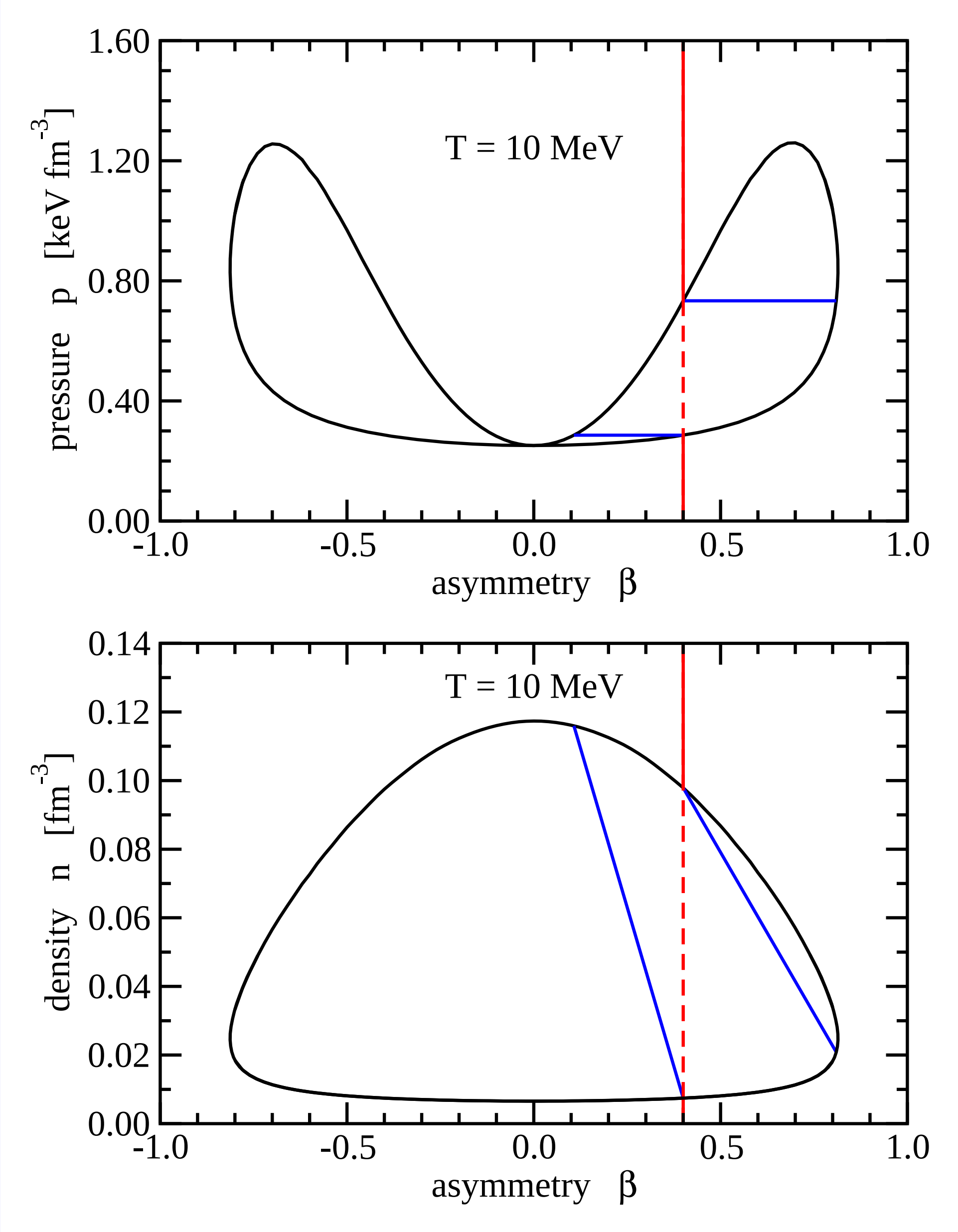
Phase Transitions III

- construction of coexisting phases I, II in equilibrium with Gibbs conditions: equal intensive variables, i.e.

$$T^{I} = T^{II} \quad p^{I} = p^{II}$$

$$\mu_b^{I} = \mu_b^{II} \quad \mu_q^{I} = \mu_q^{II}$$

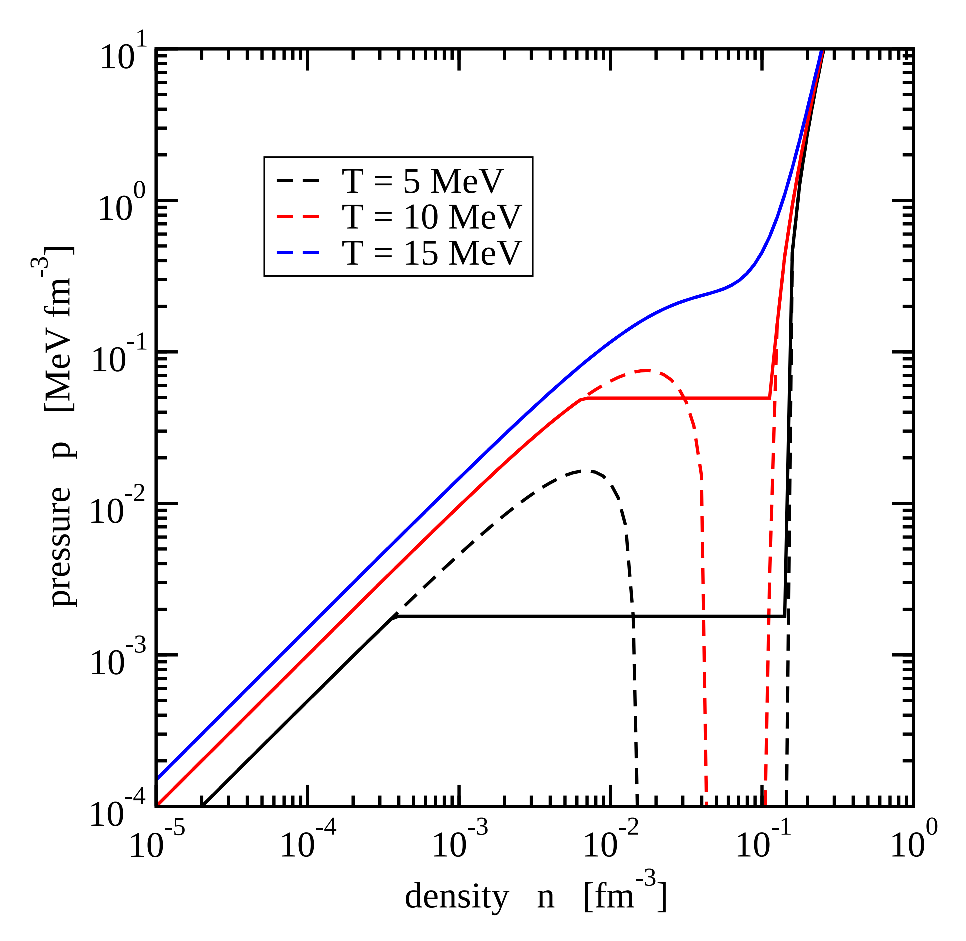
- pressure and chemical potentials not necessarily constant during phase transition
- two phases with different densities and different asymmetries
- generalization for system with electrons possible (charge neutrality)
- surface effects, Coulomb interaction
  $\Rightarrow$ inhomogeneities: heavy clusters



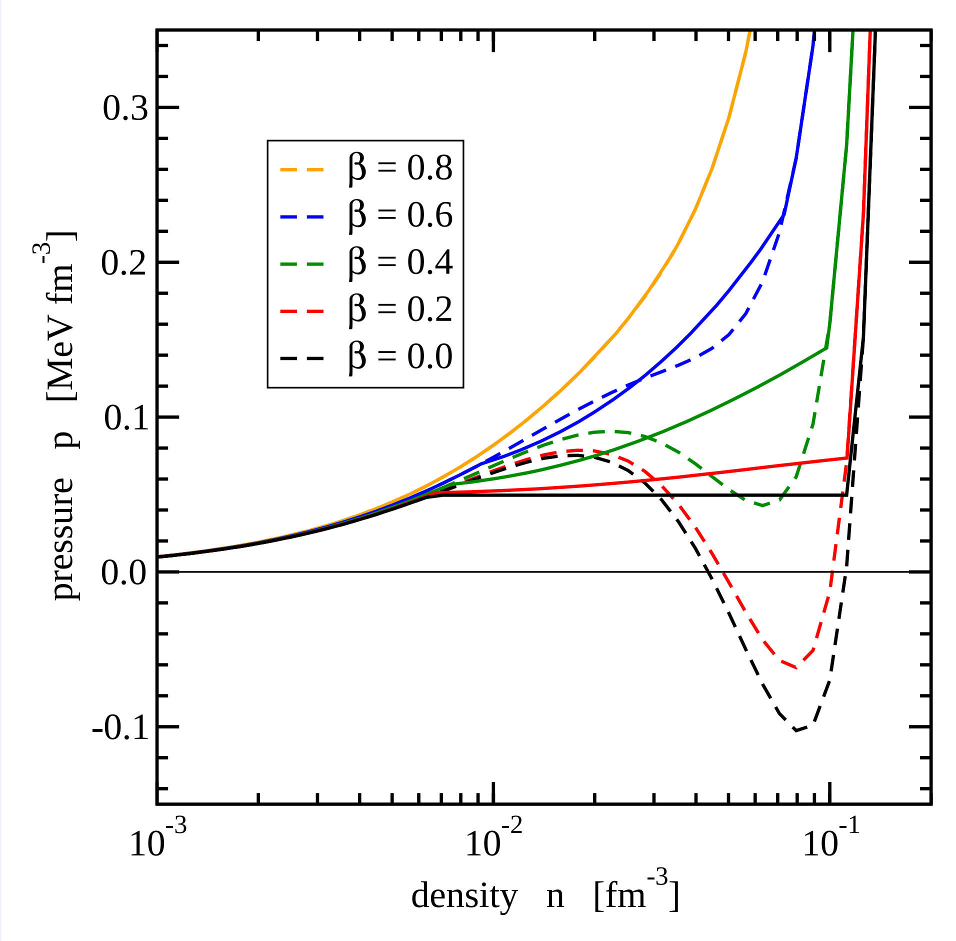
Phase Transitions IV - Pressure

without (dashed) and with (solid) phase transition

constant asymmetry $\beta = 0.0$



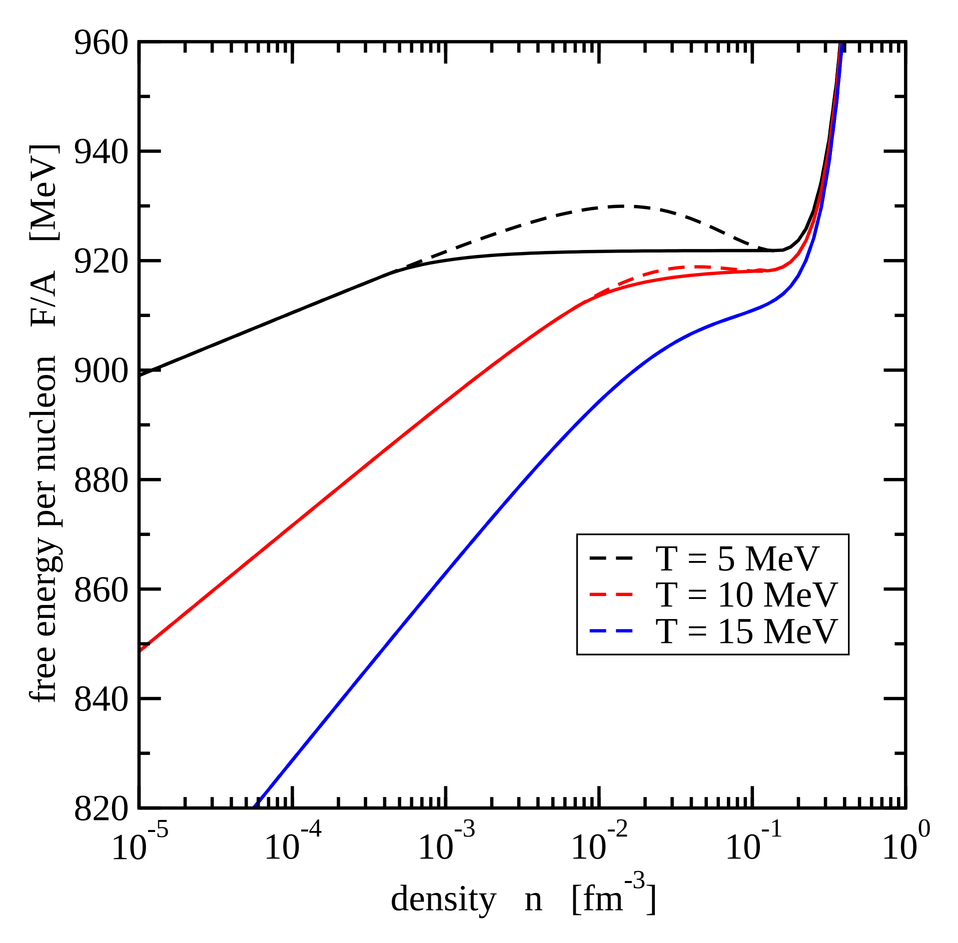
constant temperature $T = 10$ MeV



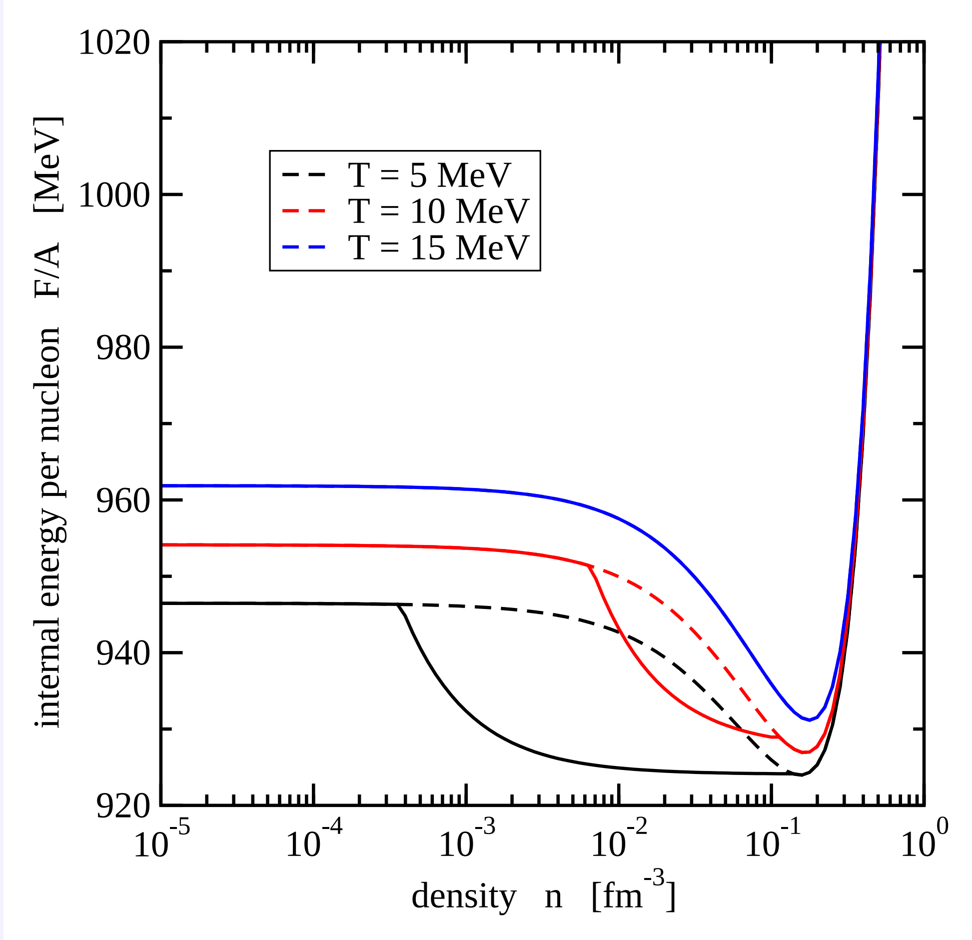
Phase Transitions V - Energies

without (dashed) and with (solid) phase transition for asymmetry $\beta = 0$

free energy per nucleon $F/A = f/n$



internal energy per nucleon $U/A = u/n$



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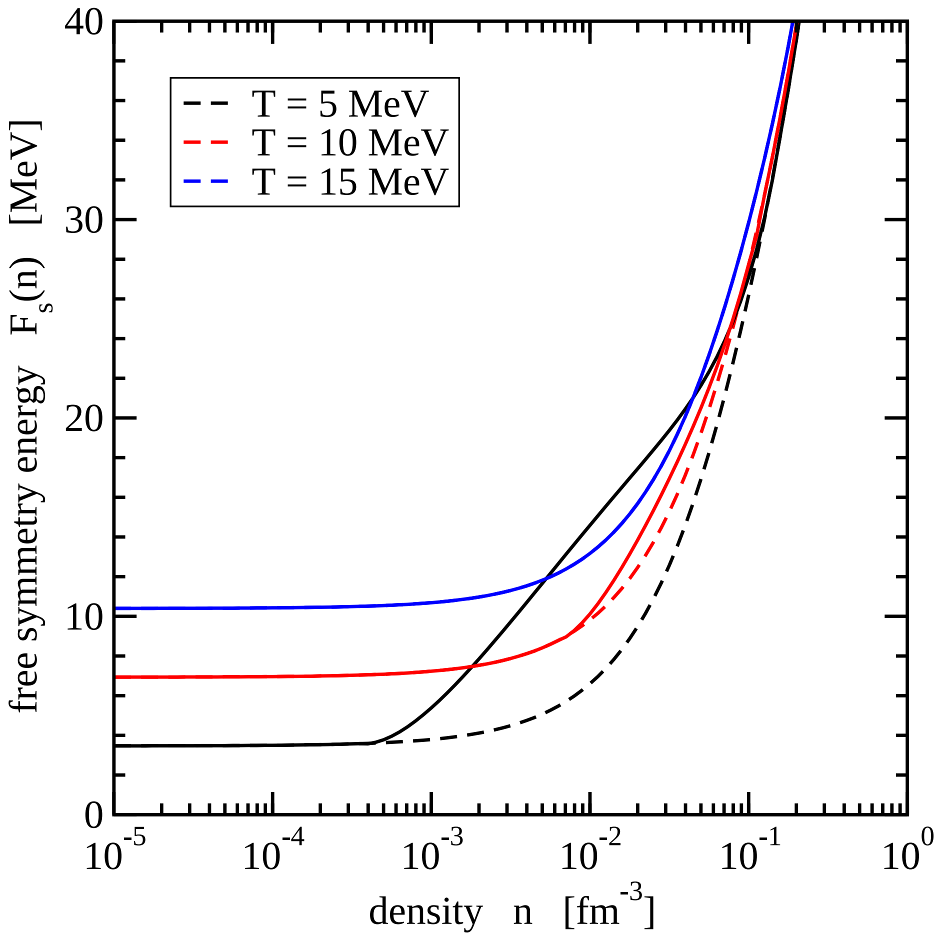
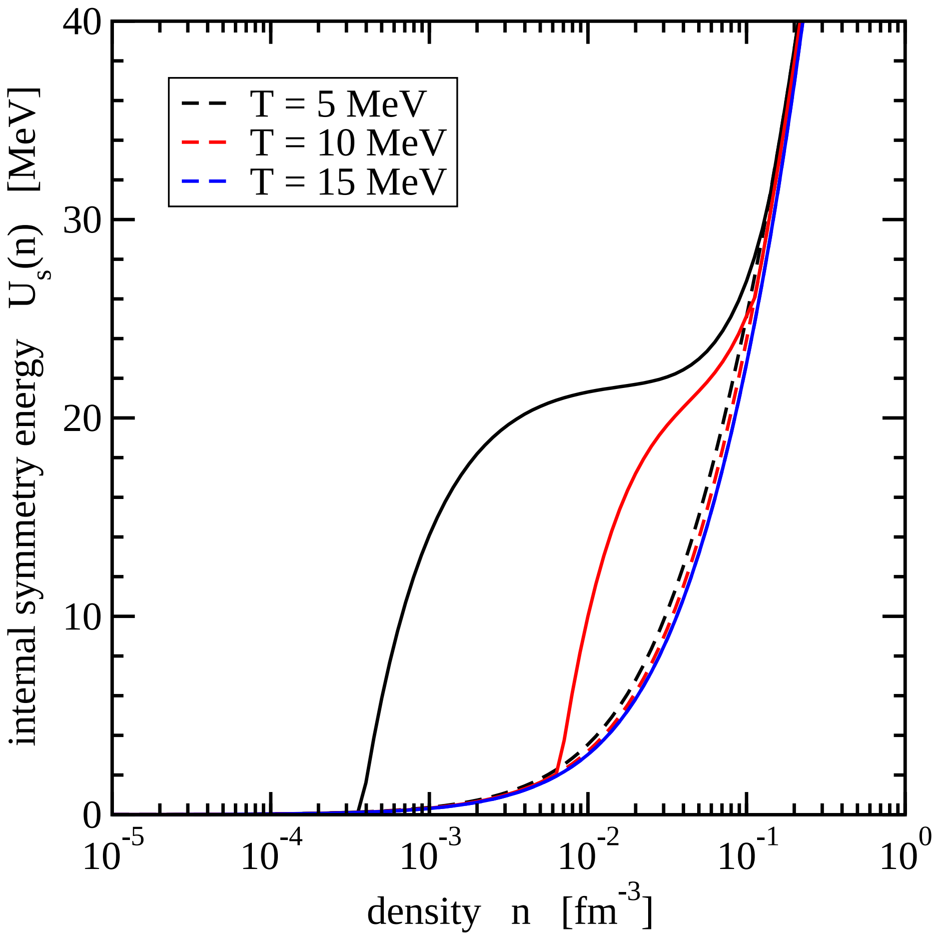
Phase Transitions VI - Symmetry Energies

without (dashed) and with (solid) phase transition

free symmetry energy



internal symmetry energy



Heavy Clusters I

low densities and low temperatures

- homogeneous system not stable $\Rightarrow$ inhomogeneities develop
- surface effects and Coulomb interaction important
- global charge neutrality $\Rightarrow$ compensation of proton charge by electron charge
- several phases (solid-liquid-gas)

description of heavy clusters

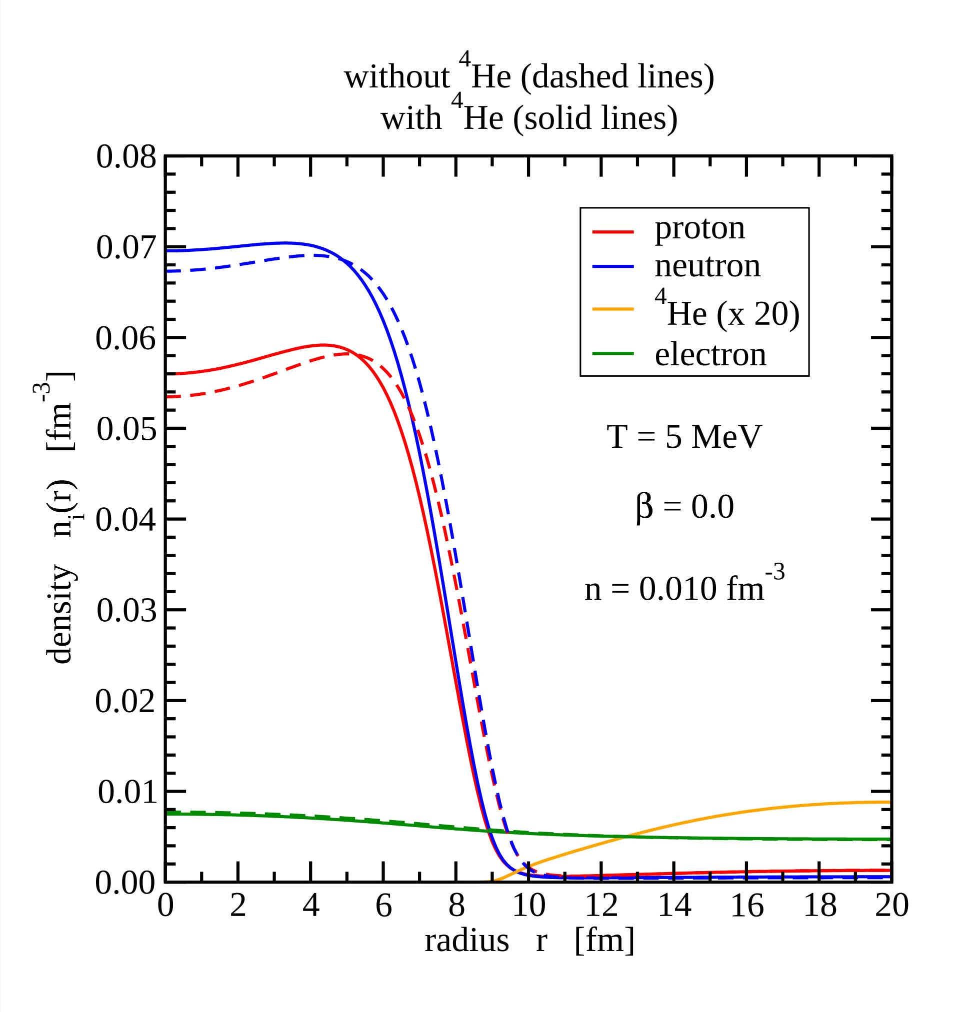
- relativistic Thomas-Fermi approximation with RMF energy density functional (beyond local density approximation)
  $\Rightarrow$ selfconsistent density distributions with smooth transition to homogeneous phase
- in gas phase: contribution of translational energy
- in solid phase/crystal: contribution of vibrational energy, Coulomb correlation energy (lattice periodic quantities)

$\Rightarrow$ modification of energy density functional before derivation of field equations etc. (violation of thermodynamical consistency if not treated correctly)

Heavy Clusters II

example

- inhomogeneous matter with protons, neutrons, electrons and α particles
- self-consistent density distributions in spherical Wigner-Seitz cell
- formation of heavy nucleus
- shell/bubble solutions possible
- distribution of α particles around nucleus (no excluded volume mechanism needed)
- effects of Coulomb interaction
- inhomogeneous electron distribution (charge screening)



Summary and Outlook

- construction of improved equation of state of dense nuclear matter
  - relativistic mean-field model with density-dependent couplings
  - generalized Beth-Uhlenbeck approach (light clusters)
  - relativistic Thomas-Fermi approximation (heavy clusters)

- various constraints of model parameters
- improved consistency

- work in progress
  - parametrization of medium effects for clusters/correlations to be investigated
  - improvement of RMF parametrization
  - only preliminary results so far, still numerical difficulties

- application (future)
  - detailed investigation of new EoS
  - effects on astrophysical models for supernovae?