Measurements and Analysis of the Transverse Beam Transfer Function (BTF) at the SIS 18 Synchrotron

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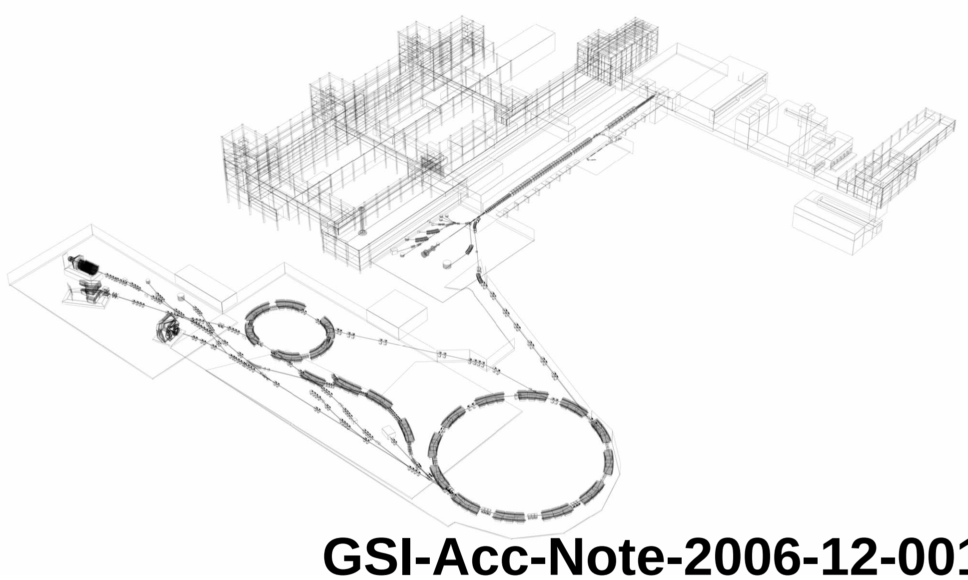

GSI-Acc-Note-2006-12-001

December 1, 2006

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Abstract

The Beam Transfer Function (BTF) belongs to the most powerful beam diagnostics methods, providing direct measurements of beam stability properties, impedances and various beam and lattice parameters. Here, we report about the first measurement of the transverse BTF and the corresponding stability diagrams in SIS 18. By comparing experimental BTF and stability diagrams with theoretical expressions, the betatron tune, chromaticity and momentum spread are determined, origin and type of Landau damping is identified.

1 Introduction

By exciting a beam with a periodic signal and measuring the resulting beam response (amplitude and phase) the Beam Transfer Function (BTF) is obtained [1], which contains a wealth of important information on beam and machine properties. In the transverse plane, it provides direct measurements of:

• beam stability properties (inversed BTF shows the stability diagram which provides threshold of instabilities);

• external impedances $Z^{\perp}(\Omega)$;

• betatron tunes, chromaticity and longitudinal momentum spread;

• damping properties of a transverse feedback system, cooler, etc.

BTF techniques have been widely employed, e.g. in the Fermilab Main Ring [2], in LEAR [3] and in COSY [4]. An important advantage is the non-destructive nature of the method: a rather weak excitation is enough and the beam operation is not disturbed.

2 Transverse Beam Transfer Function

Consider the equation of betatron oscillations,

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \omega_\beta^2 y = c_\mathrm{n}\overline{y}(t)Z^{\perp}(\Omega) + \hat{G}e^{-i\Omega t} \ , \tag{1}$$

where the first term on the right-hand side is the effect of an external impedance $Z^{\perp}(\Omega)$, representing the interaction of the beam with the surroundings. Thus this term is proportional to the transverse displacement of the center of mass $\overline{y}(t)$ (c_n is a normalization constant). The second term on the right-hand side is the exciting periodic force, which in our case is a kicker in the transverse plane. The amplitude of the driven transverse coherent oscillation $\hat{\overline{y}}$, which establishes after a long enough time interval, is the response of the beam to this excitation. Divided by the exciting force amplitude $\hat{G}$, it gives the Beam Transfer Function (BTF). Considering firstly the situation without impedances $Z^{\perp}$, we obtain the 'intrinsic' BTF,

$$R_0(\Omega) = A\frac{\hat{\overline{y}}}{\hat{G}} \ , \tag{2}$$

with A a normalization constant. For the case, where the betatron tune spread in the beam is caused e.g. by a spread in the longitudinal momentum we average Eq. (1) over all particles of the beam and obtain for an excitation around the slow wave frequency

$\omega_s = n\omega_0 - \omega_\beta$,

$$R_0(\Omega) = -\frac{A}{2\omega_\beta}\int \frac{\rho(\omega_s)\mathrm{d}\omega_s}{\omega_s - \Omega} = -\frac{A}{2\omega_\beta}\left[i\pi\rho(\Omega) + \mathrm{PV}\int \frac{\rho(\omega_s)\mathrm{d}\omega_s}{\omega_s - \Omega}\right] , \quad (3)$$

$$\text{with} \quad A = i\frac{Nq^2}{m\gamma}\frac{\omega_0}{(2\pi)^2 R} . \quad (4)$$

Taking now into account an external impedance $Z^\perp(\Omega)$, we express BTF in the simple form,

$$\frac{1}{R(\Omega)} = \frac{1}{R_0(\Omega)} - Z^\perp(\Omega) , \quad (5)$$

which shows that the measured and inversed BTF is the inversed intrinsic one, shifted by the negative value of the impedance at the same frequency. The inversed BTF, represented in the complex plane, gives the (shifted) stability diagram. Eq. (5) demonstrates that the BTF contains information about beam stability (first term on the right-hand side) and external impedances (second term on the right-hand side).

The properties of linear Landau damping and of the resulting BTF are defined by the frequency distribution $\rho(\omega_s)$ in Eq. (3) and by the frequency spread, which has the following form for the slow wave (δf^-) and for the fast wave (δf^+),

$$\frac{\delta f^-}{f_0} = \frac{\delta p}{p}\left|\eta(m - Q_f + Q\frac{\xi}{\eta})\right| , \quad (6)$$

$$\frac{\delta f^+}{f_0} = \frac{\delta p}{p}\left|\eta(m + Q_f - Q\frac{\xi}{\eta})\right| , \quad (7)$$

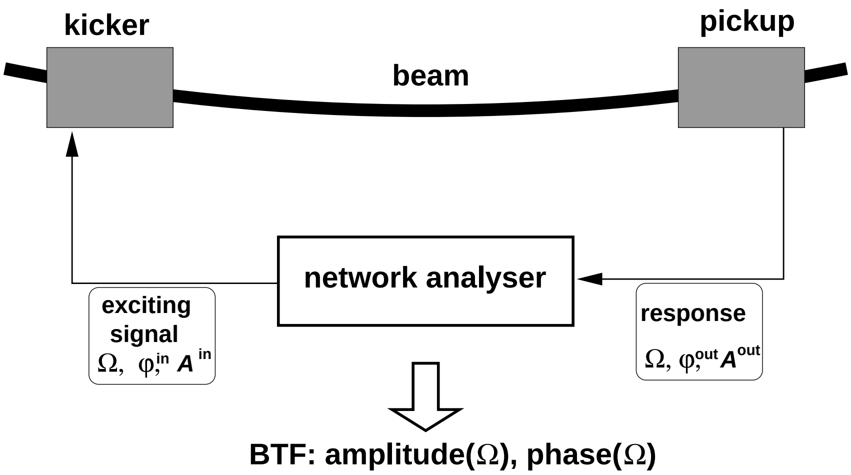

Figure 1: Principal scheme of a BTF measurement with a sweep over frequency Ω.

where Q_f is the fractional part of the tune Q, $f_0 = \omega_0/2\pi$ is the revolution frequency, $\delta p/p$ is the momentum spread, ξ is the chromaticity and is η the slip-factor. Note that m is the harmonic number, i.e. the observed signal for the corresponding wave lies near mf_0. The mode number of the related slow wave is $n = m + Q - Q_f$, and for the fast wave it is $n = m - Q + Q_f$. For the fast wave frequency $\omega_\mathrm{f} = n\omega_0 + \omega_\beta$ Eq. (3) should be used with the opposite sign.

In practice, a network analyser is used to generate the exciting signal and to compare it with the beam response, see Fig. 1 which illustrates the idea of a BTF measurement. A frequency scan over Ω is performed in order to obtain the BTF amplitude and phase as a function of frequency.

3 Results of the measurements and analysis

Measurements of the transverse (vertical only) BTF were performed in the SIS 18 synchrotron at GSI Darmstadt on October 25, 2006. A coasting beam of U^{73+} at the flat-top energy (500 MeV/u, f_0=1.048 MHz) was kept for 10 s to allow a slow frequency sweep. The typical particle number was 2.8×10^8, which corresponds to a current of 3.4 mA. The HP-8753C network analyzer was employed. As a pickup for the vertical signal the Schottky probe was used.

Figure 2 shows the measured amplitude of the beam response for the harmonic number $m = 25$. We see that the beam response is maximized at the lower- and at the upper side band frequencies. The distance between the maxima is $2Q_f f_0$, which gives the fractional tune Q_f =0.306 (the set tune was $Q_\mathrm{vert} = 3.29$). This measurement was made with a compensated chromaticity $\xi \approx 0$. Under this condition the observation that the upper side band is slightly broader and smaller then the lower one is in agreement with Eqs. (6), (7).

Further results of the BTF measurements and the yielding stability diagrams are presented in Figs. 3–6. All these signals correspond to the natural chromaticity in SIS 18. The black lines (left figures) are the raw experimental data for the BTF amplitude, the blue dashed lines are the corresponding phase signals. The black lines in the right-hand side figures are raw data for stability diagrams (the inversed BTF) in the impedance plane $V + iU = Z^\perp$.

Firstly, we analyse the BTF measured for the slow wave at the harmonic number $m = 24$, see Figs. 3, 4. In order to determine the type of the momentum distribution [$\rho(\omega_\mathrm{s})$ in Eq. (3)], we compare the experimental stability diagram with three distributions in Fig. 4. The green line corresponds to the bi-Lorentz distribution,

$$\rho(\omega) = \frac{2}{\pi\delta\omega\left[1 + \frac{(\omega-\omega_0)^2}{\delta\omega^2}\right]^2} \ , \tag{8}$$

the blue one to the parabolic distribution,

$$\rho(\omega) = \frac{3}{4\delta\omega}\left(1 - \frac{(\omega-\omega_0)^2}{\delta\omega^2}\right) \ \text{for} \ |\omega-\omega_0| \le \delta\omega \ ; \quad \rho(\omega) = 0 \ \text{otherwise} \ , \tag{9}$$

and the red line corresponds to the Gaussian distribution,

$$\rho(\omega) = \frac{1}{\sqrt{2\pi}\delta\omega}\exp\left\{-\frac{(\omega-\omega_0)^2}{2\,\delta\omega^2}\right\} \ . \tag{10}$$

From Fig. 4 it is obvious that the Gaussian distribution is the most suitable one to describe the particle momentum distribution. Also we conclude that the transverse stability properties of the beam can be attributed to linear Landau damping due to the longitudinal momentum spread $\delta p/p$.

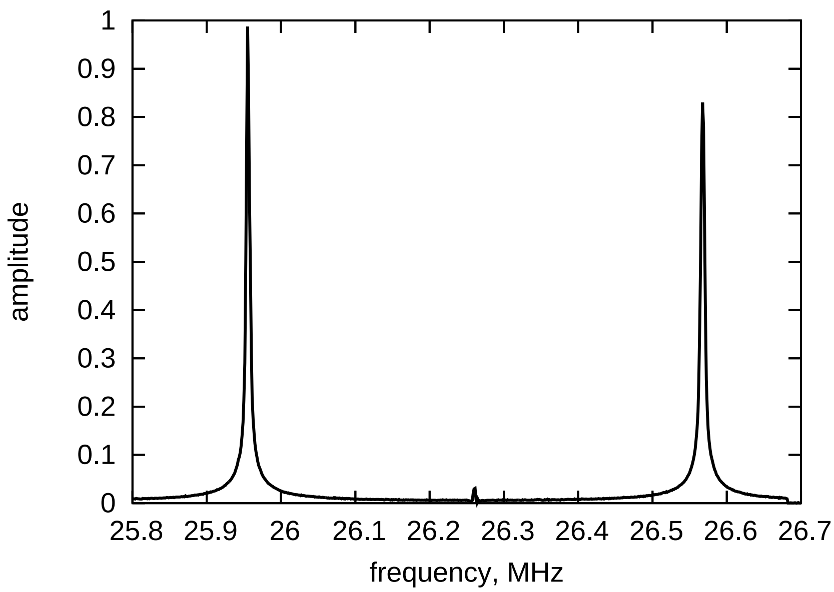

Figure 2: Amplitude of the beam response with both side bands for the harmonic number $m = 25$, $mf_0 = 26.26\,\text{MHz}$. The distance between the lines is $2Q_f f_0$, which gives for the fractional tune Q_f =0.306. The upper side band is slightly broader and smaller here, because this measurement corresponds to a compensated chromaticity $\xi \approx 0$ [see Eqs. (6), (7)].

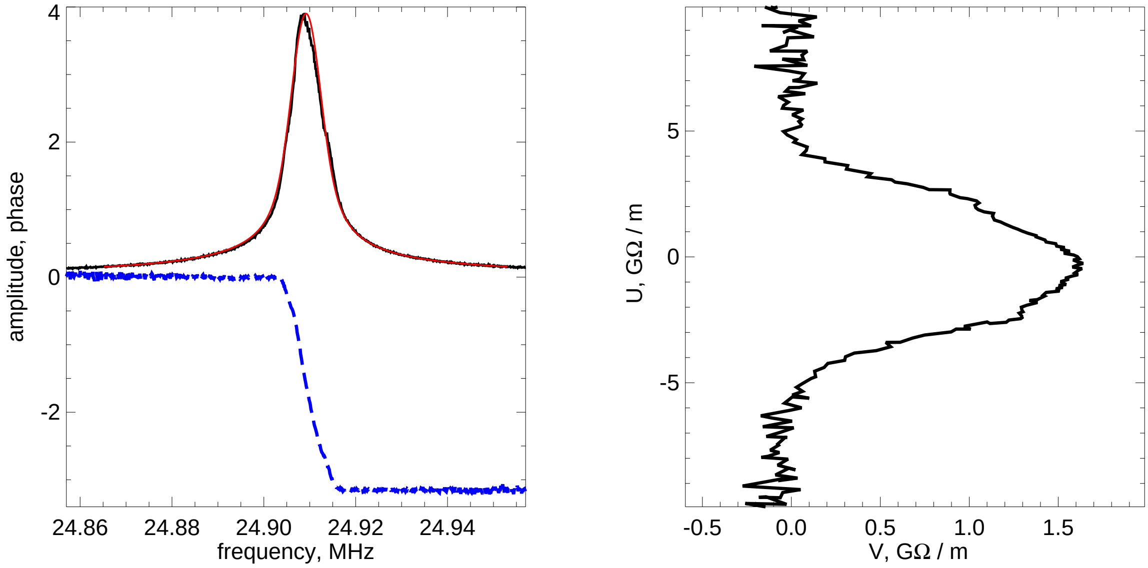

Figure 3: BTF measurement for the lower side band of $m = 24$. Left: amplitude (black line) and phase (blue dashed line) of the BTF signal, red line is the fitting for the Gaussian distribution. Right: resulting stability diagram (inversed BTF).

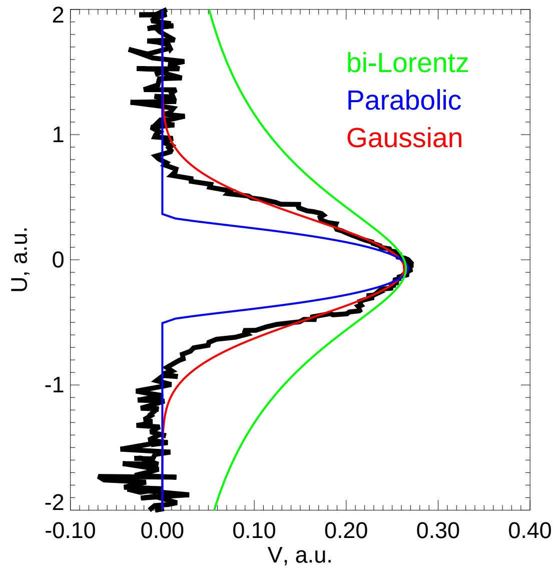

Figure 4: Measured stability diagram from Fig. 3 (black line) with theoretical stability diagrams for bi-Lorentz (green), Parabolic (blue) and Gaussian (red) distributions.

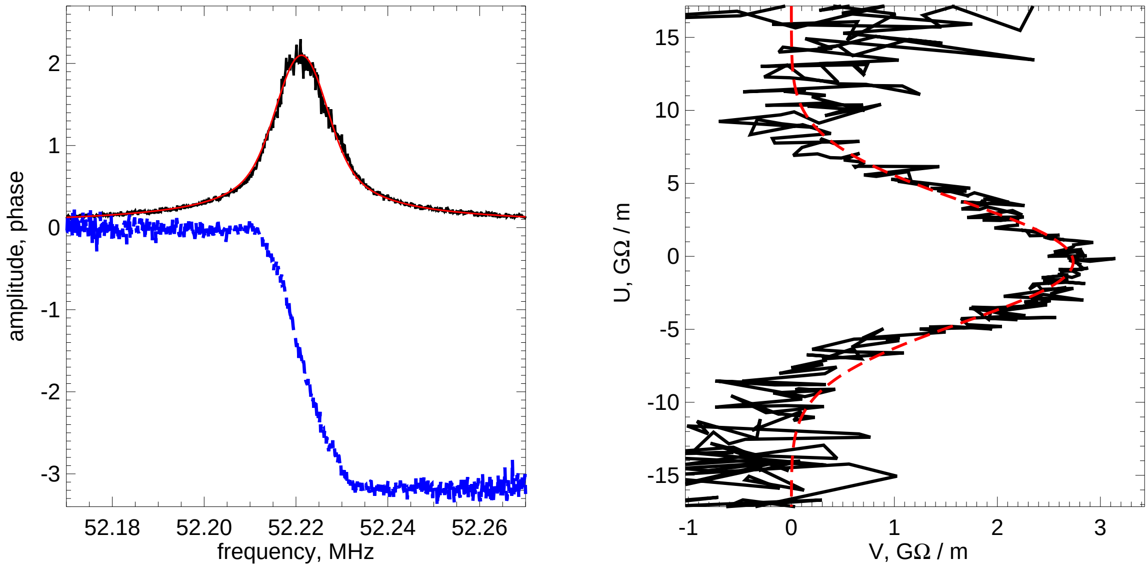

Figure 5: BTF measurement for the lower side band of $m = 50$. Left: amplitude (black line) and phase (blue dashed line) of the BTF signal, the red line is the fitting for the Gaussian distribution. Right: resulting stability diagram (black) with the theoretical one (red dashed) for the Gaussian distribution.

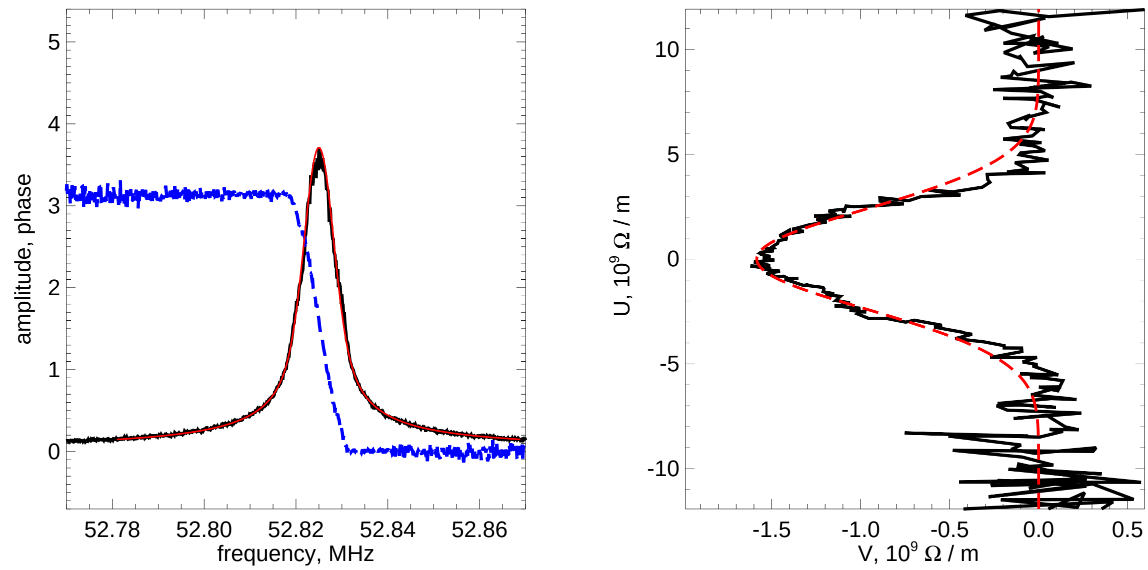

Figure 6: BTF measurement for the upper side band of $m = 50$. Notation is the same as in Fig. 5.

Using our findings from Fig. 4, we fit the BTF amplitude of the Gaussian distribution to the experimental one (the red lines in left-hand side plots). In this manner we determine the width of the corresponding side band. For example, for the lower side band at $m = 24$ (Fig. 3) we obtain $\delta f_{\rm s}$=2.2 kHz, which means that the momentum spread is $\delta p/p$=1.43×10^{-4} (which corresponds to the rms-equivalent parabolic spread 3.2×10^{-4}).

Figures 5 and 6 present measured BTF and stability diagrams for the lower and upper side band of $m = 50$. Using BTF of the both side bands for the same harmonic number one can determine the chromaticity ξ, see Eqs. (6), (7). Using our signals from Figs. 5, 6 we obtain $\xi = -1.62$. Resulting momentum spread is $\delta p/p = 1.46 \times 10^{-4}$ (rms-equivalent parabolic 3.26×10^{-4}). Note that, in contrast to Fig. 2, the lower side band is considerably broader than the upper one here, which is related to the large negative chromaticity.

4 Conclusions and Outlook

By employing the existing equipment we were able to successfully perform transverse BTF measurements in SIS 18. Stability diagrams of a sufficiently good quality were obtained. We found that the stability properties of the observed beam can be explained within the framework of linear Landau damping caused by the momentum spread. The momentum distribution is very well described by a Gaussian. From the measured BTF we obtained the betatron tune Q, momentum spread $\delta p/p$ and chromaticity ξ.

With regard to the further development, it will be advisable to firstly measure the BTF also in the horizontal plane. The horizontal signal is usually of worse quality, because of the larger distance of the pick-up plates from the beam. One important aim is to measure the impedances of the kickers and of the resistive wall. Therefore it has to be taken into account that the Schottky pick-up, used within this measurement, operates well for frequencies larger then ∼10 MHz. Concerning the kicker impedance (horizontal plane), measurements with the Schottky probe should be possible, because there are peaks of the $\mathcal{R}e(Z^{\perp})$ which are expected at e.g. 12 MHz and 15.5 MHz [5]. For the resistive wall impedance measurements at ∼200 kHz are necessary [5], thus the BPMs should be employed instead of the Schottky probe, which requires additional test measurements. Higher beam intensities (comparing with the present work) are needed for impedance measurements to resolve clearly the shift of the stability diagram. Collective effects, as nonlinear space charge, can then significantly modify BTF signals. These aspects are presently investigated numerically [6, 7].

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