

# EXTENDED HAMILTON-LAGRANGE FORMALISM AND ITS APPLICATION TO FEYNMAN'S PATH INTEGRAL FOR RELATIVISTIC QUANTUM PHYSICS

J. Struckmeier

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## **Abstract:**

We present a consistent and comprehensive treatise on the foundations of the *extended Hamilton-Lagrange formalism* - where the dynamical system is parameterized along a general system evolution parameter  $s$ , and the time  $t$  is treated as a *dependent* variable  $t(s)$  on equal footing with all other configuration space variables  $q'(s)$ . In the action principle, the conventional classical action  $Ldt$  is then replaced by the generalized action  $L_1 ds$ , with  $L$  and  $L_1$  denoting the conventional and the extended Lagrangian, respectively. It is shown that a unique correlation of  $L_1$  and  $L$  exists if we refrain from performing simultaneously a transformation of the dynamical variables. With the appropriate correlation of  $L_1$  and  $L$  in place, the extension of the formalism preserves its *canonical form*.

In the extended formalism, the dynamical system is described as a *constrained* motion within an *extended space*. We show that the value of the constraint and the parameter  $s$  constitutes an *additional pair* of canonically conjugate variables. In the corresponding quantum system, we thus encounter an additional *uncertainty relation*.

As a consequence of the formal similarity of conventional and extended Hamilton-Lagrange formalisms, Feynman's *non-relativistic* path integral approach can be converted on a *general level* into a form appropriate for *relativistic* quantum physics. In the emerging parameterized quantum description, the additional uncertainty relation serves as the means to incorporate the constraint and hence to finally eliminate the parameterization.

We derive the extended Lagrangian  $L_1$  of a classical relativistic point particle in an external electromagnetic field and show that the generalized path integral approach yields the Klein-Gordon equation as the corresponding quantum description. We furthermore derive the space-time propagator for a free relativistic particle from its extended Lagrangian  $L_1$ . These results can be regarded as the proof of principle of the *relativistic generalization* of Feynman's path integral approach to quantum physics.