Canonical transformations and exact invariants for time-dependent Hamiltonian systems

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Abstract:

An exact invariant is derived for *n*-degree-of-freedom non-relativistic Hamiltonian systems with general time-dependent potentials. To work out the invariant, an *infinitesimal* canonical transformation is performed in the framework of the extended phase-space. We apply this approach to derive the invariant for a specific class of Hamiltonian systems. For the considered class of Hamiltonian systems, the invariant is obtained equivalently performing in the extended phase-space a *finite* canonical transformation of the initially time-dependent Hamiltonian to a time-independent one. It is furthermore shown that the invariant can be expressed as an integral of an energy balance equation.
The invariant itself contains a time-dependent auxiliary function $\xi(t)$ that represents a solution of a linear third-order differential equation, referred to as the auxiliary equation. The coefficients of the auxiliary equation depend in general on the explicitly known configuration space trajectory defined by the system's time evolution. This complexity of the auxiliary equation reflects the generally involved phase-space symmetry associated with the conserved quantity of a time-dependent non-linear Hamiltonian system. Our results are applied to three examples of time-dependent damped and undamped oscillators. The known invariants for time-dependent and time-independent harmonic oscillators are shown to follow directly from our generalized formulation.

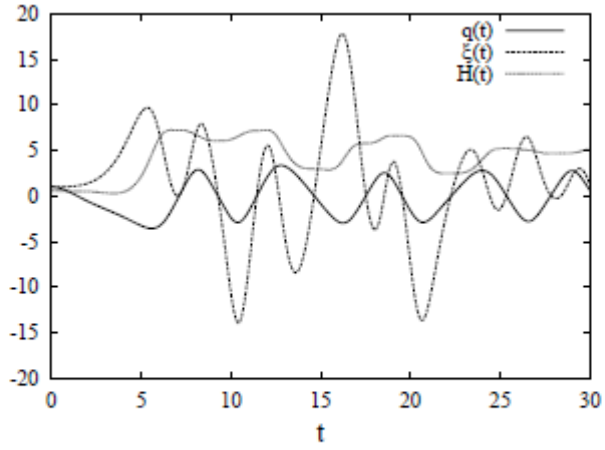

Fig. 2 Example of a simultaneous numerical integration of the equation of motion (57) and the coupled set (60), (61) for $\xi(t)$. In addition, $H(t)$ displays the time-dependent system energy given by the Hamiltonian (56).